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D-brane states and disk amplitudes in OSp invariant closed string field theory

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Abstract: We construct solitonic states in the OSp invariant string field theory, which are BRST invariant in the leading order of regularization parameter ϵ. We calculate the disk amplitudes using these solitonic states and show that they describe D-branes and ghost D-branes.

Keywords: BRST Symmetry, String Field Theory, Bosonic Strings, D-branes.

Contents

1. Introduction

D-branes have been playing a central role in string theory for a number of years. They can be considered as soliton solutions of open string field theory. For example, in bosonic open string field theory, Sen conjectured that Dp-branes with $p < 25$ are described as unstable lump solutions [1] and this was tested in many papers starting with [2 − 4]. What we would like to study in this paper is how one can realize D-branes in closed string field theory.

The closed string field theory that we consider here is the OSp invariant string field theory [5] for bosonic strings. (See also [6 − 9].) The OSp invariant string field theory is a covariantized version of the light-cone gauge string field theory [10 − 12]. It is constructed so that the S-matrix elements of the light-cone theory are reproduced by using this formulation. However an extra time variable exists in the formulation, and the action of this theory looks different from that of the usual field theory. Therefore, the OSp invariant string field theory should be considered as something like stochastic or Parisi-Sourlas type formulation of field theory [13, 14]. In our previous work [15], treating the theory in such a manner, we constructed BRST invariant observables in the OSp invariant string field theory, from which one can calculate the S-matrix elements of string theory.

In [15], only on-shell asymptotic states are considered and the observables are BRST invariant up to the nonlinear terms. In order to construct off-shell BRST invariant states, we should take the nonlinear terms into account. What we would like to do in this paper is to construct such states using boundary states for D-branes. We consider states which act as source terms in the string field theory and have the effect of generating boundaries in the worldsheet. Imposing the condition that the states are BRST invariant in the leading order of regularization parameter ϵ, we can fix the form of the states. These states can be considered as states in which there exist solitons corresponding to D-branes and ghost D-branes [16]. We can construct states with an arbitrary number of such solitons. We calculate the disk amplitudes using these states and show that the disk amplitudes of bosonic string theory in the presence of D-branes and ghost D-branes are reproduced.

In [17], solitonic states corresponding to even number of D-branes or ghost D-branes were constructed by using the similarity between the string field theory for noncritical strings [18] and the OSp invariant string field theory. Although our construction in this paper is essentially the same as that in [17], we get different results because of several reasons. Firstly, in [17], we considered the solitonic operators in analogy to noncritical string theory [19, 20] and postulated the form of the vacuum amplitude using this analogy. We checked that the vacuum amplitude coincide with that for even number of D-branes in string theory. However, the vacuum amplitude is a constant which may be changed at will by changing the definition. In this paper, we rather calculate the disk amplitudes, which can be defined without such ambiguities. We show that the treatment of the solitonic states in [17] corresponds to considering even number of solitons. Secondly, in [17], we defined the creation and the annihilation operators corresponding to normalized boundary states and performed calculations using these operators, which made the calculations rather indirect. In this paper, we do not introduce the artificial "normalized boundary states", and calculate the BRST transformation of the solitonic states directly. We find that a factor of 2 was overlooked in the calculations of [17].

The organization of this paper is as follows. In section 2, we construct solitonic states in the OSp invariant string field theory. Imposing the condition that the states are BRST invariant in the leading order of regularization parameter ϵ, we can fix the form of the states. In section 3, we calculate disk amplitudes using our solitonic states and show that disk amplitudes in the presence of D-branes and ghost D-branes are reproduced including the normalizations. Thus we identify the solitons with D-branes and ghost D-branes. Section 4 is devoted to discussions. In appendix A, we summarize the formulation of the OSp invariant string field theory. In appendices B and C, we present the details of calculations needed to show the BRST invariance of the solitonic states.

2. BRST invariant solitonic states

In this paper, the notations for the variables of the OSp invariant string field theory are the same as those used in [15], otherwise stated. Those are summarized in appendix A.

Let us consider a Dp-brane (or a ghost Dp-brane) that extends in the X^μ ($\mu = 26, 1, \ldots, p$) directions and is located at $X^i = 0$ ($i = p+1, \ldots, 25$).[1] We denote these

[1] In this paper, we consider a flat noncompact space-time. We do not need any infrared regularization in the calculations here, in contrast to those in [17].

directions by X^μ ($\mu \in \mathrm{N}$) and X^i ($i \in \mathrm{D}$), respectively.

In the OSp invariant string field theory, the boundary state $|B_0\rangle$ corresponding to the Dp-brane can be constructed as follows [17]. The matter fields $X^\mu(\tau,\sigma)$, $X^i(\tau,\sigma)$ and the ghost fields $C(\tau,\sigma)$, $\bar{C}(\tau,\sigma)$ satisfy the following boundary conditions at $\tau = 0$,

$$\partial_\tau X^\mu(0,\sigma)|B_0\rangle = 0\,, \quad X^i(0,\sigma)|B_0\rangle = 0\,, \quad C(0,\sigma)|B_0\rangle = \bar{C}(0,\sigma)|B_0\rangle = 0\ . \tag{2.1}$$

It follows that the state $|B_0\rangle$ is expressed in terms of the oscillation modes as

$$|B_0\rangle = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\alpha^N_{-n}\tilde{\alpha}^M_{-n}D_{NM}\right]|0\rangle(2\pi)^{p+1}\delta^{p+1}_{\mathrm{N}}(p)\,, \tag{2.2}$$

where $\delta^{p+1}_{\mathrm{N}}(p)$ denotes the delta function of the momentum in the directions along the Dp-brane defined as $\delta^{p+1}_{\mathrm{N}}(p) = \prod_{\mu\in\mathrm{N}}\delta(p_\mu)$, and D_{NM} denotes

$$D_{NM} = D^{NM} = \begin{array}{cc} & \begin{array}{cccc} \phantom{\delta_{\mu\nu}} & \phantom{-\delta_{ij}} & C & \bar{C} \end{array} \\ \begin{array}{c} \\ \\ C \\ \bar{C} \end{array} & \begin{pmatrix} \delta_{\mu\nu} & & & \\ & -\delta_{ij} & & \\ & & 0 & i \\ & & -i & 0 \end{pmatrix} \end{array} \qquad \text{with} \quad \mu,\nu \in \mathrm{N}\,,\ i,j \in \mathrm{D}\ . \tag{2.3}$$

Since the norm of the boundary state $|B_0\rangle$ diverges, we need to regularize it. In order to do so, we introduce

$$|B_0\rangle^T = e^{-\frac{T}{|\alpha|}(L_0+\tilde{L}_0-2)}|B_0\rangle \tag{2.4}$$

for $T > 0$, and consider $|B_0\rangle^\epsilon$ with $0 < \epsilon \ll 1$ as a regularized version of $|B_0\rangle$. Notice that the operator $e^{-\frac{T}{|\alpha|}(L_0+\tilde{L}_0-2)}$ commutes with the BRST operator Q_{B}.

2.1 States with one soliton

Using the regularized boundary state $|B_0\rangle^\epsilon$, let us construct a state in the following form in the Hilbert space of the OSp invariant string field theory,

$$|D\rangle\!\rangle \equiv \lambda\int d\zeta\,\bar{\mathcal{O}}_D(\zeta)|0\rangle\!\rangle\,, \tag{2.5}$$

where

$$\bar{\mathcal{O}}_D(\zeta) = \exp\left[a\int_{-\infty}^{0}dr\,\frac{e^{\zeta\alpha_r}}{\alpha_r}\,{}_r^\epsilon\langle B_0|\Phi\rangle_r + F(\zeta)\right]\ . \tag{2.6}$$

Here λ and a are constants, $F(\zeta)$ is a function of ζ and the limits of the zero-mode integration dr in the exponent of $\bar{\mathcal{O}}_D(\zeta)$ denotes the integration region of the string length α_r. Since the integration is over $-\infty < \alpha_r < 0$, only the creation operators contribute to $\bar{\mathcal{O}}_D(\zeta)$. Assuming that the integration over α_r is convergent with $\mathrm{Re}\,\zeta > 0$ sufficiently large, we define $\bar{\mathcal{O}}_D(\zeta)$ by analytic continuation.

Expanding the exponential in terms of the string field, it is easy to see that the state $|D\rangle\!\rangle$ has the effect of generating boundaries in the worldsheet, with a weight which depends on a and $F(\zeta)$. Let us impose the condition that the state $|D\rangle\!\rangle$ is BRST invariant in the leading order of ϵ. As we will see, we can determine a and $F(\zeta)$ from this condition.

BRST transformation. In order to evaluate $\delta_{\mathrm{B}}|D\rangle\rangle$, we should calculate the BRST transformation of the operator in the exponent of $\bar{\mathcal{O}}_D(\zeta)$:

$$\begin{aligned}\delta_{\mathrm{B}} \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|\Phi\rangle_r &= \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|Q_{\mathrm{B}}^{(r)}|\Phi\rangle_r \\ &\quad + g\int_0^{\infty} d3\, \frac{e^{-\zeta\alpha_3}}{\alpha_3} \int d1d2\, \langle V_3(1,2,3)|\Phi\rangle_1|\Phi\rangle_2|B_0\rangle_3^{\epsilon}\ . \end{aligned} \tag{2.7}$$

By using

$$Q_{\mathrm{B}}\left(\frac{1}{\alpha}|B_0\rangle^{\epsilon}\right) = 0\,, \tag{2.8}$$

one can recast the first term on the right hand side of eq. (2.7) into

$$\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|Q_{\mathrm{B}}^{(r)}|\Phi\rangle_r = \zeta \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|i\pi_0^{(r)}|\bar{\psi}\rangle_r\ . \tag{2.9}$$

Let us here introduce shorthand notations

$$\begin{aligned}\bar{\phi}(\zeta) &\equiv \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|\Phi\rangle_r\,, \\ \bar{\chi}(\zeta) &\equiv \int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|i\pi_0^{(r)}|\bar{\psi}\rangle_r\,, \end{aligned} \tag{2.10}$$

in terms of which eq. (2.6) can be expressed as

$$\bar{\mathcal{O}}_D(\zeta) = \exp\left(a\bar{\phi}(\zeta) + F(\zeta)\right)\,, \tag{2.11}$$

and eq. (2.9) can be written as

$$\int_{-\infty}^{0} dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|Q_{\mathrm{B}}^{(r)}|\Phi\rangle_r = \zeta\bar{\chi}(\zeta)\ . \tag{2.12}$$

Notice that $\bar{\phi}$ and $\bar{\chi}$ are made only from the creation modes and commute with each other.

For the second term on the right hand side of eq. (2.7), we decompose $|\Phi\rangle$ into the creation and annihilation parts as $|\Phi\rangle = |\psi\rangle + |\bar{\psi}\rangle$, and obtain

$$\begin{aligned} & g\int_0^{\infty} d3\, \frac{e^{-\zeta\alpha_3}}{\alpha_3} \int d1d2\, \langle V_3(1,2,3)|\Phi\rangle_1|\Phi\rangle_2|B_0\rangle_3^{\epsilon} \\ &= g\int_0^{\infty} d3\, \frac{e^{-\zeta\alpha_3}}{\alpha_3} \Bigg[\int_{-\infty}^{0} d1 \int_0^{\infty} d2\, \langle V_3(1,2,3)|\bar{\psi}\rangle_1|\psi\rangle_2|B_0\rangle_3^{\epsilon} \\ &\qquad + \int_0^{\infty} d1 \int_{-\infty}^{0} d2\, \langle V_3(1,2,3)|\psi\rangle_1|\bar{\psi}\rangle_2|B_0\rangle_3^{\epsilon} \\ &\qquad + \int_{-\infty}^{0} d1 \int_{-\infty}^{0} d2\, \langle V_3(1,2,3)|\bar{\psi}\rangle_1|\bar{\psi}\rangle_2|B_0\rangle_3^{\epsilon}\Bigg]\ . \end{aligned} \tag{2.13}$$

It follows from the relation $\langle V_3(1,2,3)| = \langle V_3(2,1,3)|$ that the first and the second terms on the right hand side of this equation are equal to each other.

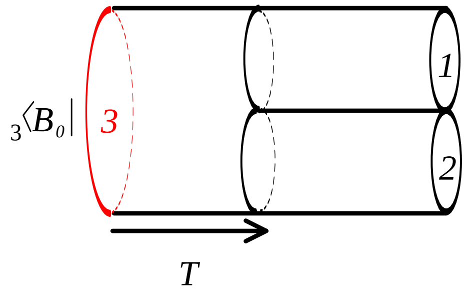

Figure 1: The string diagram corresponding to the vertex $\langle V_2(1,2);T|$.

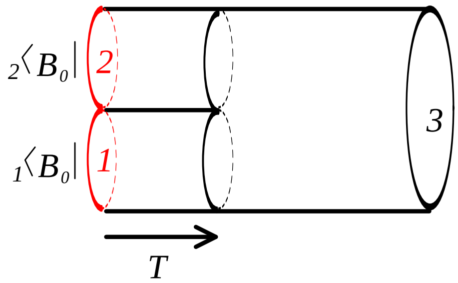

Figure 2: The string diagram corresponding to the vertex $\langle V_1(3);T|$.

In this form, it is straightforward to calculate the BRST transformation of $|D\rangle\rangle$. Using the commutation relation (A.22), we have

$$\begin{aligned}\delta_{\rm B}|D\rangle\rangle &= \lambda \int d\zeta \, \exp\left(a\bar{\phi}(\zeta) + F(\zeta)\right) \\ &\quad \times \left[a\zeta\bar{\chi}(\zeta) + ga^2 \int_{-\infty}^{0} d1 \int_{0}^{\infty} d2 \int_{0}^{\infty} d3 \frac{e^{\zeta\alpha_1}}{\alpha_2\alpha_3} \langle V_3(1,2,3)|\bar{\psi}\rangle_1 |B_0\rangle_2^{\epsilon} |B_0\rangle_3^{\epsilon} \right. \\ &\quad \left. + ga \int_{-\infty}^{0} d1 \int_{-\infty}^{0} d2 \int_{0}^{\infty} d3 \frac{e^{\zeta(\alpha_1+\alpha_2)}}{\alpha_3} \langle V_3(1,2,3)|\bar{\psi}\rangle_1 |\bar{\psi}\rangle_2 |B_0\rangle_3^{\epsilon}\right] |0\rangle\rangle \;. \end{aligned} \tag{2.14}$$

This tells us that in order to evaluate the BRST transformation of the state $|D\rangle\rangle$ we need to obtain

$$\langle V_2(1,2);T| \equiv \int d'3 \, \langle V_3(1,2,3)|B_0\rangle_3^T \tag{2.15}$$

and

$$\langle V_1(3);T| \equiv \int d'1 d'2 \, \langle V_3(1,2,3)|B_0\rangle_1^T |B_0\rangle_2^T \tag{2.16}$$

for $T = \epsilon$. Here $\alpha_1\alpha_2 > 0$ in both cases. The integration measure $d'r$ is defined in eq. (A.6). These vertices respectively correspond to the string diagrams depicted in figures 1 and 2.

By using these vertices, eq. (2.14) can be rewritten as

$$\begin{aligned}\delta_{\rm B}|D\rangle\rangle &= \lambda \int d\zeta \, \exp\left(a\bar{\phi}(\zeta) + F(\zeta)\right) \\ &\quad \times \left[a\zeta\bar{\chi}(\zeta) + \frac{ga^2}{4} \int_{0}^{\infty} d\alpha_1 \int_{0}^{\infty} d\alpha_2 \int_{-\infty}^{0} d3 e^{\zeta\alpha_3} \langle V_1(3);\epsilon|\bar{\psi}\rangle_3 \right. \\ &\quad \left. + \frac{ga}{2} \int_{-\infty}^{0} d1 \int_{-\infty}^{0} d2 \int_{0}^{\infty} d\alpha_3 e^{\zeta(\alpha_1+\alpha_2)} \langle V_2(1,2);\epsilon|\bar{\psi}\rangle_1 |\bar{\psi}\rangle_2\right] |0\rangle\rangle \;. \end{aligned} \tag{2.17}$$

In appendices B and C, these vertices are computed in the leading order of ϵ. The results are

$$\langle V_2(1,2);\epsilon| \sim 2\delta(\alpha_1+\alpha_2+\alpha_3) \times C_2 \times {}_1^{\epsilon}\langle B_0| \, {}_2^{\epsilon}\langle B_0| \left(\frac{i}{\alpha_1}\pi_0^{(1)} + \frac{i}{\alpha_2}\pi_0^{(2)}\right) \mathcal{P}_{12} \,, \tag{2.18}$$

$$\langle V_1(3);\epsilon| \sim -2\delta(\alpha_1+\alpha_2+\alpha_3) \times C_1 \times {}_3^{\epsilon}\langle B_0| \frac{2i}{\alpha_3} \pi_0^{(3)} \mathcal{P}_3 \,, \tag{2.19}$$

where

$$C_2 \equiv \frac{1}{(16\pi)^{\frac{p+1}{2}}} \frac{4}{\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}\,, \quad C_1 \equiv \frac{(4\pi^3)^{\frac{p+1}{2}}}{(2\pi)^{25}} \frac{4}{\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}\ . \tag{2.20}$$

These are the idempotency equations [21] satisfied by the boundary states in the OSp invariant string field theory. Substituting these into eq. (2.17), we obtain

$$\delta_{\mathrm{B}}|D\rangle\!\rangle = \lambda \int d\zeta \left[a\zeta\bar{\chi}(\zeta) + ga^2 C_1 \partial_\zeta \bar{\chi}(\zeta) + 2gaC_2\,\bar{\chi}(\zeta)\partial_\zeta\bar{\phi}(\zeta)\right] e^{a\bar{\phi}(\zeta)+F(\zeta)}\,|0\rangle\!\rangle\ . \tag{2.21}$$

Here we have used the following identity

$$\int_0^\infty dl_1 \int_0^\infty dl_2 e^{-\zeta_1 l_1 - \zeta_2 l_2} f(l_1+l_2) = -\frac{\tilde{f}(\zeta_1)-\tilde{f}(\zeta_2)}{\zeta_1-\zeta_2}\,, \tag{2.22}$$

where

$$\tilde{f}(\zeta) \equiv \int_0^\infty dl e^{-\zeta l} f(l)\ . \tag{2.23}$$

Now, in order to make $|D\rangle\!\rangle$ BRST invariant, we choose $F(\zeta)$ to be of the form

$$F(\zeta) = b\zeta^2\ . \tag{2.24}$$

Then the right hand side of eq. (2.21) becomes

$$\lambda \int d\zeta\ \partial_\zeta \left[\frac{a}{2b}\bar{\chi}(\zeta)\ \exp\left(a\bar{\phi}(\zeta) + b\zeta^2\right)\right] |0\rangle\!\rangle\,, \tag{2.25}$$

provided the constants a, b satisfy

$$\frac{a}{2b} = ga^2 C_1\,, \quad \frac{a^2}{2b} = 2gaC_2\ . \tag{2.26}$$

These equations have the solutions $(a, b) = \pm(A, B)$, where

$$A = \frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\,, \quad B = \frac{(2\pi)^{13}\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}{16\left(\frac{\pi}{2}\right)^{\frac{p+1}{2}}\sqrt{\pi}g}\ . \tag{2.27}$$

Therefore, by choosing (a, b) as $\pm(A, B)$ and taking the integration contour for ζ appropriately, we can obtain a state BRST invariant in the leading order of ϵ. Let us define

$$|D_\pm\rangle\!\rangle \equiv \lambda_\pm \int d\zeta\, \bar{\mathcal{O}}_{D_\pm}(\zeta)|0\rangle\!\rangle\,, \tag{2.28}$$

with

$$\bar{\mathcal{O}}_{D_\pm}(\zeta) = \exp\left[\pm\frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}} \int_{-\infty}^0 dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}^{\epsilon}_{r}\langle B_0|\bar{\psi}\rangle_r \pm \frac{(2\pi)^{13}\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}{16\left(\frac{\pi}{2}\right)^{\frac{p+1}{2}}\sqrt{\pi}g}\zeta^2\right]\ . \tag{2.29}$$

These states are considered as states in which one D-brane or one ghost D-brane is excited. We will show that $|D_\pm\rangle\!\rangle$ generate the worldsheets with boundaries with the right weight and disk amplitudes are reproduced. In this paper, we take $g > 0$. In this convention, as we will see later, $|D_+\rangle\!\rangle$ corresponds to the D-brane and $|D_-\rangle\!\rangle$ corresponds to the ghost D-brane.

One comment is in order. Here and in the following, we construct BRST invariant ket vectors in the second quantized Hilbert space. It is obvious that the hermitian conjugates of these ket vectors are also BRST invariant. Therefore the states $\langle\!\langle D_\pm|$ are BRST invariant.

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2.2 States with N solitons

We can construct BRST invariant states with N solitons in a similar way. Let us consider a state in the following form

$$|D_{N+}\rangle\!\rangle \equiv \lambda_{N+} \int \prod_{i=1}^{N} d\zeta_i \, \bar{\mathcal{O}}_{D_{N+}}(\zeta_1, \cdots, \zeta_N)|0\rangle\!\rangle \,, \tag{2.30}$$

where

$$\bar{\mathcal{O}}_{D_{N+}}(\zeta_1, \cdots, \zeta_N) = \exp\left[\sum_{i=1}^{N} \left(A\bar{\phi}(\zeta_i) + B\zeta_i^2\right) + F_N(\zeta_1, \cdots, \zeta_N)\right] . \tag{2.31}$$

Here the coefficients A and B are given in eq. (2.27), and the function $F_N(\zeta_1, \cdots, \zeta_N)$ is to be determined.

It is now straightforward to evaluate the BRST variation of this state:

$$\begin{aligned} \delta_{\mathrm{B}}|D_{N+}\rangle\!\rangle \; = \lambda_{N+} \int \prod_{i=1}^{N} d\zeta_i \exp\left[\sum_{i=1}^{N} \left(A\bar{\phi}(\zeta_i) + B\zeta_i^2\right) + F_N(\zeta_1, \cdots, \zeta_N)\right] \\ \times \left[\sum_{i=1}^{N} \Big(A\zeta_i\bar{\chi}(\zeta_i) + gA^2C_1\partial_{\zeta_i}\bar{\chi}(\zeta_i) + 2gAC_2\bar{\chi}(\zeta_i)\partial_{\zeta_i}\bar{\phi}(\zeta_i)\Big) \right. \\ \left. + gA^2C_1 \sum_{i\neq j} \frac{\bar{\chi}(\zeta_i) - \bar{\chi}(\zeta_j)}{\zeta_i - \zeta_j} \right] |0\rangle\!\rangle \; . \end{aligned} \tag{2.32}$$

Using eq. (2.26), one can easily deduce that the right hand side of eq. (2.32) can be recast into the form

$$\lambda_{N+} \int \prod_{i=1}^{N} d\zeta_i \sum_{j=1}^{N} \partial_{\zeta_j} \left[\frac{A}{2B}\bar{\chi}(\zeta_j) \exp\left\{\sum_{i=1}^{N} \left(A\bar{\phi}(\zeta_i) + B\zeta_i^2\right) + F_N(\zeta_1, \cdots, \zeta_N)\right\}\right] |0\rangle\!\rangle \,, \tag{2.33}$$

provided $F_N(\zeta_1, \cdots, \zeta_N)$ satisfies

$$\partial_{\zeta_i} F_N(\zeta_1, \ldots, \zeta_N) = \sum_{j\neq i} \frac{2}{\zeta_i - \zeta_j} \; . \tag{2.34}$$

Thus we get

$$F_N(\zeta_1, \cdots, \zeta_N) = 2\sum_{i>j} \ln(\zeta_i - \zeta_j) \,, \tag{2.35}$$

and

$$|D_{N+}\rangle\!\rangle = \lambda_{N+} \int \prod_{i=1}^{N} d\zeta_i \, \triangle_N^2 \, (\zeta_1, \cdots, \zeta_N) \exp\left[\sum_{i=1}^{N} \left(A\bar{\phi}(\zeta_i) + B\zeta_i^2\right) |0\rangle\!\rangle \right] . \tag{2.36}$$

Here $\triangle_N$ is the Vandermonde determinant.

Notice that the integration measure

$$\prod_{i=1}^{N} d\zeta_i \, \triangle_N^2 \, (\zeta_1, \cdots, \zeta_N) \tag{2.37}$$

coincides with that of the matrix models. This is natural if we regard ζ as the constant mode of tachyon on the D-brane. Since α can be considered as the length of the string, ζ may be identified with a constant tachyon mode on the boundary [17]. When there exist N D-branes, the tachyon field becomes a matrix and we should consider ζ_i as its eigenvalues. Therefore we here encounter a matrix model of constant tachyons.

$|D_{N+}\rangle\!\rangle$ can be considered as a state with N D-branes. We can also construct a state with N D-branes and M ghost D-branes as

$$|D_{N+,M-}\rangle\!\rangle \equiv \lambda_{N+,M-} \int \prod_{i=1}^{N} d\zeta_i \prod_{\bar{\imath}=1}^{M} d\zeta_{\bar{\imath}} \prod_{i>j} (\zeta_i-\zeta_j)^2 \prod_{\bar{\imath}>\bar{\jmath}} (\zeta_{\bar{\imath}}-\zeta_{\bar{\jmath}})^2 \prod_{i,\bar{\jmath}} (\zeta_i-\zeta_{\bar{\jmath}})^{-2}$$
$$\times \exp\left[A\left(\sum_{i=1}^{N} \bar{\phi}(\zeta_i) - \sum_{\bar{\imath}=1}^{M} \bar{\phi}(\zeta_{\bar{\imath}}) \right) + B\left(\sum_{i=1}^{N} \zeta_i^2 - \sum_{\bar{\imath}=1}^{M} \zeta_{\bar{\imath}}^2 \right) \right] |0\rangle\!\rangle \ . \quad (2.38)$$

This time the integration measure is that of the supermatrix model.

Before closing this section, one comment is in order. It is possible to express the state $|D_{N+,M-}\rangle\!\rangle$ as

$$|D_{N+,M-}\rangle\!\rangle \propto \left(\int d\zeta\, \mathcal{V}_{D_+}(\zeta) \right)^N \left(\int d\zeta'\, \mathcal{V}_{D_-}(\zeta') \right)^M |0\rangle\!\rangle \ . \quad (2.39)$$

Here $\mathcal{V}_{D_\pm}(\zeta)$ are of the form

$$\mathcal{V}_{D_\pm}(\zeta) = \bar{\mathcal{O}}_{D_\pm}(\zeta)\mathcal{O}_{D_\pm}(\zeta) \,, \quad (2.40)$$

where $\bar{\mathcal{O}}_{D_\pm}(\zeta)$ are the operators given in eq. (2.29) and $\mathcal{O}_{D_\pm}(\zeta)$ are defined as

$$\mathcal{O}_{D_\pm}(\zeta) = \exp\left[\pm \int_0^\infty dr\, \frac{e^{\zeta\alpha_r}}{\alpha_r}\, {}_r\langle v|\psi\rangle_r \right] , \quad (2.41)$$

with $|v\rangle$ satisfying

$$\int d'r\, {}_r\langle v|B_0\rangle_r^\epsilon = -\frac{4}{A} \ . \quad (2.42)$$

$\mathcal{V}_{D_\pm}(\zeta)$ look like vertex operators and may be considered as creation operators of D-branes and ghost D-branes. $|v\rangle$ can be any state as long as it satisfies eq. (2.42). For example, $|v\rangle$ can be taken to be proportional to $|B_0\rangle^\epsilon$ as in [17].

3. Disk amplitudes

Now that we have BRST invariant observables made from the boundary states, we would like to calculate the scattering amplitudes involving these operators and show that the amplitudes involving D-branes are reproduced. In particular, we would like to calculate the disk amplitudes in this paper.

3.1 Three-point S-matrix elements

Before going into the calculation of the disk amplitudes, it is instructive to recall how usual three-point S-matrix elements can be calculated in the OSp invariant string field theory [15]. Actually the calculation of the disk amplitudes goes in the same way as that for the three-point amplitudes. We also write down the space-time low energy effective action of the OSp invariant string field theory, which will be used to check the normalization and the sign of the amplitudes involving D-branes.

The S-matrix elements can be deduced from the correlation functions of the BRST invariant observables of the form [15]

$$\mathcal{O}(t,k) = \int dr\, \frac{1}{\alpha_r}\, {}_r\Big({}_{C,\bar{C}}\langle 0| \otimes {}_X\langle \text{primary}; k|\Big)|\Phi(t)\rangle_r\,, \tag{3.1}$$

where ${}_{C,\bar{C}}\langle 0|$ and ${}_X\langle\text{primary};k|$ denote the BPZ conjugates of the Fock vacuum $|0\rangle_{C,\bar{C}}$ in the $(C,\bar{C})$ sector and a Virasoro primary state $|\text{primary};k\rangle_X$ in the X^μ sector ($\mu=1,\ldots,26$) of the Hilbert space for the worldsheet theory, respectively. Here k_μ is the momentum eigenvalue of the state $|\text{primary};k\rangle_X$:

$$\begin{aligned} |\text{primary};k\rangle_X &= |\overline{\text{primary}}\rangle_X (2\pi)^{26}\delta^{26}(p-k)\,, \\ {}_X\langle\text{primary};k| &= (2\pi)^{26}\delta^{26}(p+k)\,{}_X\langle\overline{\text{primary}}|\,. \end{aligned} \tag{3.2}$$

$|\overline{\text{primary}}\rangle_X$ denotes the non-zero mode part of $|\text{primary};k\rangle_X$, and we normalize it as

$${}_X\langle\overline{\text{primary}}|\overline{\text{primary}}\rangle_X = 1\,. \tag{3.3}$$

The mass M of the particle corresponding to the operator $\mathcal{O}(t,k)$ can be read off from the relation

$$\left(L_0+\tilde{L}_0-2\right)|\text{primary};k\rangle_X\otimes|0\rangle_{C,\bar{C}} = \left(k^2+2i\pi_0\bar{\pi}_0+M^2\right)|\text{primary};k\rangle_X\otimes|0\rangle_{C,\bar{C}}\,. \tag{3.4}$$

Since we consider correlation functions, the primary states introduced here are off-shell in general, i.e. $k^2+M^2\neq 0$. For later use, we introduce the on-shell primary states $|\text{primary};\mathbf{k}\rangle_X = |\text{primary};k\rangle_X|_{k^2+M^2=0}$, where $\mathbf{k}$ denotes the spatial 25-momentum.

By using the canonical commutation relation (A.22), the lowest order contribution of the three-point correlation function for the observables $\mathcal{O}_r(t_r,k_r)$ $(r=1,2,3)$ $(t_1>t_2>t_3)$ with mass M_r is evaluated as

$$\begin{aligned} &\left\langle\!\!\left\langle \mathcal{O}_1(t_1,k_1)\mathcal{O}_2(t_2,k_2)\mathcal{O}_3(t_3,k_3)\right\rangle\!\!\right\rangle \\ &= \left[\int_{t_3}^{t_2} dT \prod_{s=1}^{2}\left(-\int_{-\infty}^{0}\frac{d\alpha_s}{2}\right)\int_0^\infty \frac{d\alpha_3}{2} + \int_{t_2}^{t_1} dT\left(-\int_{-\infty}^{0}\frac{d\alpha_1}{2}\right)\prod_{s=2}^{3}\left(\int_0^\infty\frac{d\alpha_s}{2}\right)\right] \\ &\quad\times 4ig\prod_{r'=1}^{3}\left(\int\frac{d^{26}p_{r'}}{(2\pi)^{26}}\, i d\bar{\pi}_0^{(r')} d\pi_0^{(r')}\right) \\ &\qquad\times\left\langle V_3^0(1,2,3)\right|\prod_{r=1}^{3}\left[e^{-i\frac{|T-t_r|}{|\alpha_r|}\left(p_r^2+M_r^2+2i\pi_0^{(r)}\bar{\pi}_0^{(r)}\right)}\left(|\text{primary}_r;k_r\rangle_X\otimes|0\rangle_{C,\bar{C}}\right)_r\right]. \end{aligned} \tag{3.5}$$

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We can readily integrate over $\alpha_3, \pi_0^{(3)}, \bar{\pi}_0^{(3)}, p_1$ and p_2, using the delta functions. In order to obtain the S-matrix elements, we need to look for the on-shell poles for the external momenta. The singular behavior at $k_2^2 + M_2^2 = 0$ comes from the region $\alpha_2 \sim 0$ in the integration over α_2 [15]. Therefore we should consider the limit $\alpha_2 \to 0$ in the three-string vertex $\langle V_3^0(1,2,3)|$. In this limit, the complicated expression (3.5) involving three Hilbert spaces for strings 1, 2 and 3 can be simply described in terms of the vertex operator as follows:

$$\begin{aligned}\Big\langle\!\!\Big\langle & \mathcal{O}_1(t_1,k_1)\mathcal{O}_2(t_2,k_2)\mathcal{O}_3(t_3,k_3)\Big\rangle\!\!\Big\rangle \\ &\sim \frac{1}{k_2^2+M_2^2}\,4ig\int_{t_3}^{t_1}dT\int_{-\infty}^{0}\frac{d\alpha_1}{2\alpha_1}\int id\bar{\pi}_0^{(1)}d\pi_0^{(1)}\,\frac{1}{\alpha_1}\,e^{-i\frac{t_1-T}{-\alpha_1}\left(k_1^2+M_1^2+2i\pi_0^{(1)}\bar{\pi}_0^{(1)}\right)} \\ &\quad\times e^{-i\frac{T-t_3}{-\alpha_1}\left(k_3^2+M_3^2+2i\pi_0^{(1)}\bar{\pi}_0^{(1)}\right)}\int\frac{d^{26}p}{(2\pi)^{26}}\,{}_X\langle \mathrm{primary}_1;k_1|\mathcal{V}_2(\mathbf{k}_2)|\mathrm{primary}_3;k_3\rangle_X\,, \end{aligned} \tag{3.6}$$

where $\mathcal{V}_2(\mathbf{k}_2)$ denotes the vertex operator corresponding to the primary state $|\mathrm{primary}_2;\mathbf{k}_2\rangle_X$ on the mass-shell associated with the observable $\mathcal{O}_2$. After the integration over $\pi_0^{(1)}$ and $\bar{\pi}_0^{(1)}$, eq. (3.6) becomes

$$\begin{aligned} &= \frac{1}{k_2^2+M_2^2}(-4g)\int_0^\infty dT'\int_0^\infty dT'' e^{-iT'\left(k_1^2+M_1^2\right)}e^{-iT''\left(k_3^2+M_3^2\right)} \\ &\qquad\qquad \times\int\frac{d^{26}p}{(2\pi)^{26}}\,{}_X\langle \mathrm{primary}_1;k_1|\mathcal{V}_2(\mathbf{k}_2)|\mathrm{primary}_3;k_3\rangle_X \\ &\sim \prod_{r=1}^{3}\left(\frac{1}{k_r^2+M_r^2}\right)4g\int\frac{d^{26}p}{(2\pi)^{26}}\,{}_X\langle \mathrm{primary}_1;\mathbf{k}_1|\mathcal{V}_2(\mathbf{k}_2)|\mathrm{primary}_3;\mathbf{k}_3\rangle_X\,. \end{aligned} \tag{3.7}$$

Here we have changed the integration variables from T and α_1 to T' and T'', where $T' = \frac{t_1-T}{-\alpha_1}$ and $T'' = \frac{T-t_3}{-\alpha_1}$. Carrying out the Wick rotation to make the space-time signature Lorentzian, we can see that the lowest order contribution to the S-matrix element for this process is

$$S = 4ig\int\frac{d^{26}p}{(2\pi)^{26}}\,{}_X\langle \mathrm{primary}_1;\mathbf{k}_1|\mathcal{V}_2(\mathbf{k}_2)|\mathrm{primary}_3;\mathbf{k}_3\rangle_X\,. \tag{3.8}$$

In following subsections, we will discuss the normalization and the sign of the disk amplitudes. In doing so, we need the space-time low energy effective action for the tachyon $T(x)$ and the graviton $h_{\mu\nu}(x)$. Let us calculate the S-matrix elements for processes involving only tachyons and gravitons. The primary states corresponding to these particles are

$$|\mathrm{primary}_r;k_r\rangle_X = \begin{cases} |0\rangle_X\,(2\pi)^{26}\delta^{26}(p-k_r) & \text{for the tachyon} \\ e_{r,\mu\nu}(k_r)\alpha_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0\rangle_X\,(2\pi)^{26}\delta^{26}(p-k_r) & \text{for the graviton} \end{cases}, \tag{3.9}$$

where $|0\rangle_X$ denotes the Fock vacuum for the X^μ sector and $e_{r,\mu\nu}(k_r)$ denotes the polarization of the asymptotic graviton state with momentum $k_{r,\mu}$. The polarization $e_{r,\mu\nu}(k_r)$ satisfies the following relations:

$$e_{r,\mu\nu} = e_{r,\nu\mu}\,,\quad \eta^{\mu\nu}e_{r,\mu\nu} = 0\,,\quad k_r^\mu e_{r,\mu\nu} = 0\,,\quad e_{r,\mu\nu}\,e_r^{\mu\nu} = 1\,. \tag{3.10}$$

The vertex operators appearing in eq. (3.6) are

$$\mathcal{V}_r(\mathbf{k}_r) = \circ\!\!\circ\, e^{ik_{r,\mu}X^\mu}(0) \,\circ\!\!\circ \tag{3.11}$$

for the tachyon and

$$\begin{aligned}\mathcal{V}_r(\mathbf{k}_r) &= -e_{r,\mu\nu}(k_r) \,\circ\!\!\circ\, \partial X^\mu \bar{\partial} X^\nu e^{ik_{r,\lambda}X^\lambda}(0) \,\circ\!\!\circ \\ &= e_{r,\mu\nu}(k_r) \,\circ\!\!\circ \left(p^\mu + \sum_{n\neq 0} \alpha_n^\mu\right)\left(p^\nu + \sum_{m\neq 0} \tilde{\alpha}_m^\nu\right) e^{ik_{r,\lambda}X^\lambda(0)} \,\circ\!\!\circ\end{aligned} \tag{3.12}$$

for the graviton. In these equations, $\circ\!\!\circ \; \circ\!\!\circ$ denotes the normal ordering of the oscillators and 0 in the arguments of the operators indicates the origin $(\tau, \sigma) = (0, 0)$ of the worldsheet.

Plugging eqs. (3.9), (3.11) and (3.12) into eq. (3.8), we obtain three-point S-matrix elements for tachyons and gravitons:

$$\begin{aligned}S_{TTT} &= 4ig\,(2\pi)^{26}\delta^{26}(k_1 + k_2 + k_3)\,, \\ S_{TTh} &= ig\, e_{3,\mu\nu} k_{12}^\mu k_{12}^\nu\,(2\pi)^{26}\delta^{26}\,(k_1 + k_2 + k_3)\ , \\ S_{hhh} &= ig\, e_{1,\mu\nu} e_{2,\alpha\beta} e_{3,\gamma\delta}\, T^{\mu\alpha\gamma} T^{\nu\beta\delta}\,(2\pi)^{26}\delta^{26}(k_1 + k_2 + k_3)\,,\end{aligned} \tag{3.13}$$

where the subscripts T and h denote the tachyon and the graviton respectively and

$$\begin{aligned}k_{rs}^\mu &= k_r^\mu - k_s^\mu\,, \\ T^{\mu\alpha\gamma} &= \eta^{\mu\alpha} k_{12}^\gamma + \eta^{\alpha\gamma} k_{23}^\mu + \eta^{\gamma\mu} k_{31}^\alpha + \frac{1}{4} k_{23}^\mu k_{31}^\alpha k_{12}^\gamma\ .\end{aligned} \tag{3.14}$$

Eq. (3.13) coincide with the results in the light-cone gauge string field theory.

We can reproduce the results obtained in eq. (3.13) from the following space-time effective action for the metric $G_{\mu\nu}(x)$ and the tachyon field $T(x)$,

$$\begin{aligned}S = \frac{1}{2\kappa^2}\int d^{26}x\sqrt{-G}R + \int d^{26}x\sqrt{-G}\left(-\frac{1}{2}G^{\mu\nu}\partial_\mu T\partial_\nu T + T^2 + \frac{2g}{3}T^3\right) \\ +\text{higher derivative terms}\,,\end{aligned} \tag{3.15}$$

by expanding the metric $G_{\mu\nu}(x)$ around the flat metric $\eta_{\mu\nu}$ as

$$G_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)\ . \tag{3.16}$$

We find that the gravitational coupling constant κ is related to the string coupling g as

$$\kappa = 2g\ . \tag{3.17}$$

3.2 Disk amplitudes

Now let us turn to the disk amplitudes. We evaluate the disk amplitude with two external closed string tachyons in the presence of one soliton, as an example. We show that our results coincide with those for a (ghost) D-brane in string theory. Using these disk amplitudes, we determine which of the states $|D_\pm\rangle\!\rangle$ corresponds to the D-brane.

Since $|D_\pm\rangle\rangle$ is a BRST invariant state, we may be able to calculate the amplitudes involving D-branes by starting from the correlation function

$$\langle\langle 0|\mathrm{T}\mathcal{O}_1(t_1)\cdots\mathcal{O}_N(t_N)|D_\pm\rangle\rangle\ . \tag{3.18}$$

Indeed, from $|D_\pm\rangle\rangle$ we get insertions of the boundary states and the worldsheets with boundaries are generated. However, because the formulation of the theory is similar to the light-cone field theory, we cannot generate the worldsheets without any external line insertions by considering

$$\langle\langle 0|D_\pm\rangle\rangle\ . \tag{3.19}$$

Such vacuum amplitudes are constants. Especially the cylinder amplitudes are constants which do not depend even on the coupling constant g. Therefore they can be considered to be included in the definition of the unknown constant $\lambda_\pm$. If we replace the bra $\langle\langle 0|$ by $\langle\langle D_\pm|$ in eq. (3.19), we get worldsheets without any external line insertions. This is calculated in [17]. But the result is a constant and cannot be distinguished from $\lambda_\pm$.

In order to normalize the correlation function (3.18), we divide it by the vacuum amplitude as in the usual field theory, and consider

$$\left\langle\left\langle \mathcal{O}_1(t_1)\cdots\mathcal{O}_N(t_N)\right\rangle\right\rangle_{D_\pm} = \frac{\langle\langle 0|\mathrm{T}\mathcal{O}_1(t_1)\cdots\mathcal{O}_N(t_N)|D_\pm\rangle\rangle}{\langle\langle 0|D_\pm\rangle\rangle}\ . \tag{3.20}$$

Therefore, starting from this normalized correlation function, we can calculate the amplitudes in the usual way.

Now let us calculate correlation functions for two closed string tachyons in the presence of the soliton, to obtain the S-matrix elements. The correlation function to be calculated is

$$\left\langle\left\langle \mathcal{O}^T_1(t_1,k_1)\mathcal{O}^T_2(t_2,k_2)\right\rangle\right\rangle_{D_\pm} = \frac{\langle\langle 0|\mathcal{O}^T_1(t_1,k_1)\mathcal{O}^T_2(t_2,k_2)|D_\pm\rangle\rangle}{\langle\langle 0|D_\pm\rangle\rangle}\ . \tag{3.21}$$

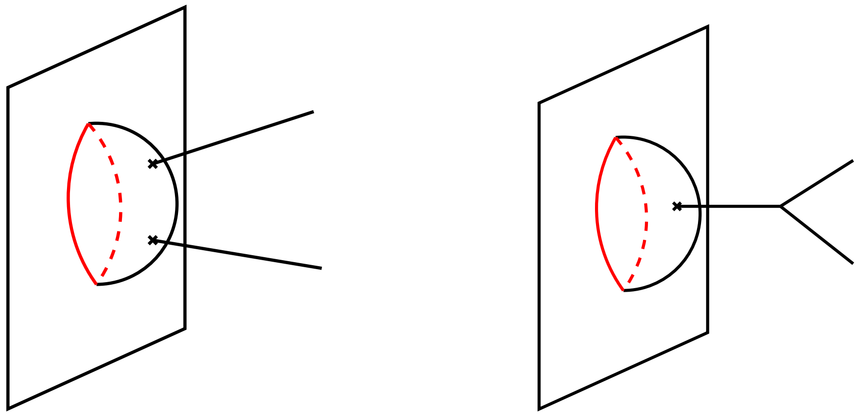
Here $\mathcal{O}^T_r$ is the observable corresponding to the tachyon state, and $t_1 > t_2$. The lowest order contributions to this correlation function give the propagator and tadpole for the tachyon. The $\mathcal{O}(g)$ term is what we should look at.

In perturbation theory, $|D_\pm\rangle\rangle$ can be recast into a more tractable form as follows. In the integrand (2.29) of the integration (2.28), the factor

$$\exp\left[\pm\frac{(2\pi)^{13}\epsilon^2\left(-\ln\epsilon\right)^{\frac{p+1}{2}}}{16\left(\frac{\pi}{2}\right)^{\frac{p+1}{2}}\sqrt{\pi}g}\zeta^2\right] \tag{3.22}$$

becomes the most dominant perturbatively. Therefore, we carry out the saddle point approximation to obtain

$$|D_\pm\rangle\rangle \simeq \lambda'_\pm \exp\left[\pm\frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\int_{-\infty}^{0}\frac{dr}{\alpha_r}{}^{\epsilon}_{r}\langle B_0|\bar{\psi}\rangle_r\right]|0\rangle\rangle\ , \tag{3.23}$$



(a) (b)

Figure 3: (a) The worldsheet diagram of the two-tachyon disk amplitudes. (b) The worldsheet diagram that contributes to the pole of intermediate closed string states.

where $\lambda'_\pm$ is given as

$$\lambda'_\pm \equiv \sqrt{\mp \frac{16\left(\frac{\pi}{2}\right)^{\frac{p+1}{2}} \pi^{\frac{3}{2}} g}{(2\pi)^{13}\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}}\,\lambda_\pm\,. \tag{3.24}$$

Notice that for $|D_+\rangle\!\rangle$ the exponent of the Gaussian factor (3.22) has the wrong sign, which makes the factor in front of λ_+ in eq. (3.24) pure imaginary. This is a sign of instability.

Then the $\mathcal{O}(g)$ term can be given as

$$\begin{aligned}
&G_{TTD_\pm}(k_1,k_2) \\
&= \left[\int_{t_3}^{t_2} dT \prod_{s=1}^{2}\left(-\int_{-\infty}^{0}\frac{d\alpha_s}{2}\right)\left(\int_0^\infty \frac{d\alpha_3}{2}\right) + \int_{t_2}^{t_1} dT\left(-\int_{-\infty}^{0}\frac{d\alpha_1}{2}\right)\prod_{s=2}^{3}\left(\int_0^\infty\frac{d\alpha_s}{2}\right)\right] \\
&\quad \times \frac{\pm 4ig(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\prod_{r'=1}^{3}\left(\int\frac{d^{26}p_{r'}}{(2\pi)^{26}} i d\bar{\pi}_0^{(r')} d\pi_0^{(r')}\right)\langle V_3^0(1,2,3)| \\
&\quad \times \prod_{r=1}^{2}\left(e^{-i\frac{|T-t_r|}{|\alpha_r|}\left(p_r^2+2i\pi_0^{(r)}\bar{\pi}_0^{(r)}-2\right)}|0\rangle_r(2\pi)^{26}\delta^{26}(p_r-k_r)\right)e^{-i\frac{T-t_3}{\alpha_3}\left(L_0^{(3)}+\tilde{L}_0^{(3)}-2\right)}|B_0\rangle_3^\epsilon\,,
\end{aligned} \tag{3.25}$$

where t_3 ($< t_1, t_2$) is the proper time of the solitonic state. In what follows, we will show that this correctly provides the contribution of the disk attached to the (ghost) D-brane corresponding to our solitonic states $|D_\pm\rangle\!\rangle$. The worldsheet diagram of this process is depicted in figure 3(a).

Eq. (3.25) is quite similar to eq. (3.5) and can be calculated in the same way. Looking for the singular behavior at $k_2^2-2=0$, we can get

$$
\begin{aligned}
&G_{TTD_\pm}(k_1,k_2) \\
&\sim \frac{1}{(k_2^2-2)}\frac{\pm 4ig(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\int_{t_3}^{t_1} dT\int_{-\infty}^{0}\frac{d\alpha_1}{2\alpha_1}\int id\bar{\pi}_0^{(1)}d\pi_0^{(1)}\,\frac{1}{\alpha_1}\,e^{-i\frac{T-t_1}{-\alpha_1}\left(k_1^2+2i\pi_0^{(1)}\bar{\pi}_0^{(1)}-2\right)} \\
&\quad\times\int\frac{d^{26}p}{(2\pi)^{26}}(2\pi)^{26}\delta^{26}(p+k_1)\,{}_X\langle 0|\,{}^{\circ}_{\circ}\,e^{ik_{2,\mu}X^\mu}(0)\,{}^{\circ}_{\circ}\,e^{-i\frac{T-t_3}{-\alpha_1}\left(L_0^X+\tilde{L}_0^X+2i\pi_0^{(1)}\bar{\pi}_0^{(1)}-2\right)}|B_0\rangle_X \\
&=\frac{1}{(k_2^2-2)}\frac{\pm 4ig(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\,i\int_0^\infty dT'\int_0^\infty dT''e^{-iT'\left(k_1^2-2\right)} \\
&\quad\times\int\frac{d^{26}p}{(2\pi)^{26}}(2\pi)^{26}\delta^{26}(p+k_1)_X\langle 0|\,{}^{\circ}_{\circ}\,e^{ik_{2,\mu}X^\mu}(0)\,{}^{\circ}_{\circ}\,e^{-iT''\left(L_0^X+\tilde{L}_0^X-2\right)}|B_0\rangle_X \\
&\sim\frac{1}{k_1^2-2}\frac{1}{k_2^2-2}\frac{\pm 4ig(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}} \\
&\quad\times\int\frac{d^{26}p}{(2\pi)^{26}}(2\pi)^{26}\delta^{26}(p+k_1)_X\langle 0|\,{}^{\circ}_{\circ}\,e^{ik_{2,\mu}X^\mu}(0)\,{}^{\circ}_{\circ}\,\frac{-i}{L_0^X+\tilde{L}_0^X-2}|B_0\rangle_X\,,
\end{aligned}
\tag{3.26}
$$

where L_0^X and $\tilde{L}_0^X$ are the zero-modes of the Virasoro generators and $|B_0\rangle_X$ is the boundary state in the X^μ sector, respectively:

$$
\begin{aligned}
L_0^X &= \frac{1}{2}p^2+\sum_{\mu\in \mathrm{N,D}}\sum_{n=1}^{\infty}\alpha_{-n}^{\mu}\alpha_{n\mu}\,,\quad \tilde{L}_0^X=\frac{1}{2}p^2+\sum_{\mu\in \mathrm{N,D}}\sum_{n=1}^{\infty}\tilde{\alpha}_{-n}^{\mu}\tilde{\alpha}_{n\mu}\,, \\
|B_0\rangle_X &= \exp\left[-\sum_{\mu,\nu\in\mathrm{N,D}}\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}^{\mu}\tilde{\alpha}_{-n}^{\nu}D_{\mu\nu}\right]|0\rangle_X(2\pi)^{p+1}\delta_{\mathrm{N}}^{p+1}(p)\,.
\end{aligned}
\tag{3.27}
$$

Carrying out the Wick rotation, we find that the S-matrix element for this process is

$$
S_{TTD_\pm}=\frac{\pm 4ig(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}}\int\frac{d^{26}p}{(2\pi)^{26}}(2\pi)^{26}\delta^{26}(p+k_1)_X\langle 0|\,{}^{\circ}_{\circ}\,e^{ik_{2,\mu}X^\mu}(0)\,{}^{\circ}_{\circ}\,\frac{1}{L_0^X+\tilde{L}_0^X-2}|B_0\rangle_X\,,
\tag{3.28}
$$

where the momenta $k_{r,\mu}$ $(r=1,2)$ are subject to the on-shell condition for the tachyon: $k_r^2=2$. It is clear that the amplitude is proportional to the usual disk amplitude.

It is straightforward to generalize the above calculations for other closed string states, just by replacing the state and the vertex operator. Also it is quite obvious that we can reproduce the disk amplitudes with more than two external lines. In order to consider the situation in which there are more than one solitons, we should replace $|D_\pm\rangle\!\rangle$ by $|D_{N+,M-}\rangle\!\rangle$. The leading order contribution in perturbation theory is from $\zeta_i=\zeta_{\bar{\imath}}=0$ in eq. (2.38) and we obtain the S-matrix element as S_{TTD_+} in eq. (3.28) multiplied by $N-M$.

We can also replace the bra $\langle\!\langle 0|$ in eq. (3.21) by the solitonic states. By doing so, we introduce more solitons and it is easy to see that the disk amplitudes are multiplied by the total number of D-branes minus that of ghost D-branes. Therefore, it is now clear that we considered situations with even number of solitons in [17], by taking the bra and the ket to be hermitian conjugate to each other. In this paper, considering that the vacuum amplitudes are included in the definitions of $\lambda_\pm$, we can realize more general situations.

3.3 D-brane and ghost D-brane states

Let us check if the disk amplitude (3.28) has the correct normalization. At the on-shell pole of an intermediate closed string state $|\text{primary}; k\rangle_X$, it is factorized as

$$S_{TTD_\pm} \sim \int \frac{d^{26}k}{(2\pi)^{26}} \left[4ig \int \frac{d^{26}p'}{(2\pi)^{26}} (2\pi)^{26} \delta^{26}(p'+k_1)_X \langle 0| \circ e^{ik_{2,\mu}X^\mu}(0) \circ |\text{primary}; k\rangle_X \right] \times \frac{-i}{k^2+M^2} \times \left[\frac{\pm i (2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}} \int \frac{d^{26}p}{(2\pi)^{26}} {}_X\langle \text{primary}; -k|B_0\rangle_X \right], \quad (3.29)$$

where M denotes the mass of the state. Since the D-brane can be considered as a source of closed string states, the low energy effective action should have source terms at $x^i = 0$ ($i \in$ D) due to the presence of solitons. From eq. (3.29), we can read off the source terms as

$$S'_\pm = \pm \frac{(2\pi)^{13}}{(8\pi^2)^{\frac{p+1}{2}}\sqrt{\pi}} \int d^{26}x \prod_{i\in \mathrm{D}} \delta(x^i) \left[T(x) - 2 \sum_{\mu,\nu \in \mathrm{N}} h_{\mu\nu}(x)\eta^{\mu\nu} + \cdots \right], \quad (3.30)$$

where the ellipsis denotes the contribution from the states other than the tachyon $T(x)$ and the graviton $h_{\mu\nu}(x)$. This can be compared with the DBI action for a flat Dp-brane located at $x^i = 0$ ($i \in$ D):

$$S_p = -\tau_p \int d^{26}x \prod_{i\in \mathrm{D}} \delta(x^i) \sqrt{-\det_{\mu,\nu\in \mathrm{N}} G_{\mu\nu}(x)}\,, \quad (3.31)$$

where τ_p is the Dp-brane tension in bosonic string theory defined as [22, 23][2]

$$\tau_p = \frac{\sqrt{\pi}}{16\kappa} (8\pi^2)^{\frac{11-p}{2}}\ . \quad (3.32)$$

Using eq. (3.16) we can expand S_p in terms of $h_{\mu\nu}(x)$, and obtain the source term for $h_{\mu\nu}(x)$ which coincides with that in S'_+ in eq. (3.30). Therefore the disk amplitude S_{TTD_+} coincides with that for a D-brane and S_{TTD_-} coincides with that for a ghost D-brane.

Hence we should identify $|D_+\rangle\rangle$ with the state with one D-brane and $|D_-\rangle\rangle$ with the state with one ghost D-brane. This identification is quite consistent. D-branes in bosonic string theory are unstable due to the lack of the RR-charge and the soliton corresponding to the state $|D_+\rangle\rangle$ is also unstable, as was mentioned below eq. (3.24).

4. Discussion

In this paper, we construct solitonic states corresponding to D-branes and ghost D-branes and check that the disk amplitudes coincide with the usual string theory results. These solitonic states are BRST invariant in the leading order of ϵ. Since the BRST variation in eq. (2.25) is of order $\epsilon^{-2}(-\ln\epsilon)^{-\frac{p+1}{2}}$, higher order corrections do not go to 0 in the limit $\epsilon \to 0$. For $p \neq -1$, the correction terms are of order $\epsilon^{-2}(-\ln\epsilon)^{-\frac{p+1}{2}-n}$ ($n > 0$) and

[2] In this paper, we use the units in which $\alpha' = 2$.

for $p = -1$, the next leading term is of order ϵ^0. It might be possible to prove that by modifying the exponent of $\bar{\mathcal{O}}_{D\pm}(\zeta)$ as

$$\exp\left[\pm A\bar{\phi}(\zeta) \pm B\zeta^2 + (\text{terms higher order in } \epsilon)\right] , \tag{4.1}$$

it becomes BRST invariant. As is clear from the calculation of the disk amplitudes, the higher order terms do not contribute to the amplitudes in the limit $\epsilon \to 0$. Of course, we need to examine the form of the BRST transformation to show that this actually happens. We do not try doing so, because here we are dealing with bosonic strings and we are destined to have insurmountable divergences any way. Hopefully, we may be able to show the BRST invariance more completely in the superstring case.

The calculation of the disk amplitudes goes in the same way as that in the usual amplitudes. Open string external lines may be introduced by deforming the boundary state by the marginal operators corresponding to the open string vertex operators. It is an intriguing problem to examine if the higher order open string amplitudes are reproduced correctly. Another problem is to calculate the open string amplitudes without closed string insertions.

The variables ζ in the definition of the solitonic states can be regarded as constant tachyon. They are conjugate to the α in the OSp invariant string field theory. Therefore somehow a part of the open string modes is incorporated in the formulation of the closed string field theory. It may be possible to generalize this to other modes of open strings.

Acknowledgments

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A. *OSp* invariant string field theory

In this appendix, we summarize the formulation of the OSp invariant string field theory.

variables. The coordinate variables on the worldsheet in the OSp invariant string field theory are the $OSp(26|2)$ vector $X^M = \left(X^\mu, C, \bar{C}\right)$, where X^μ ($\mu = 1, \ldots, 26$) are Grassmann even and the ghost fields C and $\bar{C}$ are Grassmann odd. The metric of the $OSp(26|2)$ vector space is

$$\eta_{MN} = \begin{array}{c c} & \begin{array}{cc} \phantom{\delta_{\mu\nu}} & C \;\; \bar{C} \end{array} \\ \begin{array}{c} \\ \\ C \\ \bar{C} \end{array} & \left(\begin{array}{c|cc} \delta_{\mu\nu} & & \\ \hline & 0 & -i \\ & i & 0 \end{array}\right) \end{array} = \eta^{MN} , \tag{A.1}$$

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where we have taken the Euclidean signature for the physical space-time. X^M are Fourier expanded in the usual way and we obtain the non-zero oscillation modes

$$\begin{aligned} \alpha_n^M &= (\alpha_n^\mu, -\gamma_n, \bar{\gamma}_n) \ , \\ \tilde{\alpha}_n^M &= (\tilde{\alpha}_n^\mu, -\tilde{\gamma}_n, \bar{\tilde{\gamma}}_n) \quad (n \neq 0) \, , \end{aligned} \tag{A.2}$$

and the zero modes

$$\begin{aligned} x^M &= \left(x^\mu, C_0, \bar{C}_0\right) \ , \\ \alpha_0^M &= \tilde{\alpha}_0^M = p^M = (p^\mu, -\pi_0, \bar{\pi}_0) \ . \end{aligned} \tag{A.3}$$

They satisfy the canonical commutation relations

$$[x^N, p^M\} = i\eta^{NM} \, , \quad [\alpha_n^N, \alpha_m^M\} = n\eta^{NM}\delta_{n+m,0} \, , \quad [\tilde{\alpha}_n^N, \tilde{\alpha}_m^M\} = n\eta^{NM}\delta_{n+m,0} \tag{A.4}$$

for $n, m \neq 0$, where the graded commutator $[A, B\}$ denotes the anti-commutator when A and B are both fermionic operators and the commutator otherwise.

We define the Fock vacuum $|0\rangle$ in the usual way and take the momentum representation for the wave functions for the zero modes. The integration measure for the zero-modes of the r-th string is defined as

$$dr \equiv \frac{\alpha_r d\alpha_r}{2} \frac{d^{26}p_r}{(2\pi)^{26}} \, id\bar{\pi}_0^{(r)} d\pi_0^{(r)} \ . \tag{A.5}$$

It is convenient to define the measure $d'r$ for $p_r^\mu, \pi_0^{(r)}, \bar{\pi}_0^{(r)}$ as

$$d'r = \frac{d^{26}p_r}{(2\pi)^{26}} id\bar{\pi}_0^{(r)} d\pi_0^{(r)} \ . \tag{A.6}$$

action. The action of the OSp invariant string field theory takes the form

$$\begin{aligned} S = \int dt \Bigg[&\frac{1}{2} \int d1d2 \, \langle R(1,2) \, |\Phi\rangle_1 \left(i\frac{\partial}{\partial t} - \frac{L_0^{(2)} + \tilde{L}_0^{(2)} - 2}{\alpha_2} \right) |\Phi\rangle_2 \\ &+ \frac{2g}{3} \int d1d2d3 \, \langle V_3^0(1,2,3) | \, \Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \Bigg] \ . \end{aligned} \tag{A.7}$$

Here $\langle R(1,2)|$ is the reflector given as

$$\langle R(1,2)| = \delta(1,2) \; {}_{12}\langle 0| \, e^{E(1,2)} \, \frac{1}{\alpha_1} \, , \tag{A.8}$$

where

$$\begin{aligned} {}_{12}\langle 0| &= {}_1\langle 0|{}_2\langle 0| \, , \\ E(1,2) &= -\sum_{n=1}^{\infty} \frac{1}{n} \left(\alpha_n^{N(1)} \alpha_n^{M(2)} + \tilde{\alpha}_n^{N(1)} \tilde{\alpha}_n^{M(2)} \right) \eta_{NM} \, , \\ \delta(1,2) &= 2\delta(\alpha_1 + \alpha_2)(2\pi)^{26} \delta^{26}(p_1 + p_2) i(\bar{\pi}_0^{(1)} + \bar{\pi}_0^{(2)})(\pi_0^{(1)} + \pi_0^{(2)}) \ . \end{aligned} \tag{A.9}$$

$\langle V_3^0(1,2,3)|$ is the three-string vertex given as

$$\left\langle V_3^0(1,2,3)\right| \equiv \delta(1,2,3)\ {}_{123}\langle 0|e^{E(1,2,3)}\mathcal{P}_{123}\frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3}\,, \tag{A.10}$$

where

$$\begin{aligned}
{}_{123}\langle 0| &= {}_1\langle 0|\,{}_2\langle 0|\,{}_3\langle 0|\,,\\
\mathcal{P}_{123} &= \mathcal{P}_1\mathcal{P}_2\mathcal{P}_3\,,\quad \mathcal{P}_r = \int_0^{2\pi}\frac{d\theta}{2\pi}e^{i\theta\left(L_0^{(r)}-\tilde{L}_0^{(r)}\right)}\,,\\
\delta(1,2,3) &= 2\delta\Big(\sum_{s=1}^{3}\alpha_s\Big)(2\pi)^{26}\delta^{26}\Big(\sum_{r=1}^{3}p_r\Big)\,i\Big(\sum_{r'=1}^{3}\bar{\pi}_0^{(r')}\Big)\Big(\sum_{s'=1}^{3}\pi_0^{(s')}\Big)\,,\\
E(1,2,3) &= \frac{1}{2}\sum_{n,m\geq 0}\sum_{r,s=1}^{3}\bar{N}^{rs}_{nm}\left(\alpha_n^{N(r)}\alpha_m^{M(s)}+\tilde{\alpha}_n^{N(r)}\tilde{\alpha}_m^{M(s)}\right)\eta_{NM}\,,\\
\mu(1,2,3) &= \exp\left(-\hat{\tau}_0\sum_{r=1}^{3}\frac{1}{\alpha_r}\right)\,,\quad \hat{\tau}_0 = \sum_{r=1}^{3}\alpha_r\ln|\alpha_r|\ .
\end{aligned} \tag{A.11}$$

Here $\bar{N}^{rs}_{nm}$ denote the Neumann coefficients associated with the joining-splitting type of three-string interaction [10−12]. g is the coupling constant for strings. In this paper, we take $g>0$.

The string field Φ is taken to be Grassmann even and subject to the level matching condition $\mathcal{P}\Phi=\Phi$ and the reality condition

$$\langle\Phi_{\rm hc}| = \langle\Phi|\ . \tag{A.12}$$

Here $\langle\Phi_{\rm hc}|\equiv(|\Phi\rangle)^\dagger$ denotes the hermitian conjugate of $|\Phi\rangle$, and $\langle\Phi|$ denotes the BPZ conjugate of $|\Phi\rangle$ defined as

$${}_2\langle\Phi| = \int d1\,\langle R(1,2)|\Phi\rangle_1\ . \tag{A.13}$$

We also define

$$|R(1,2)\rangle \equiv \delta(1,2)\frac{1}{\alpha_1}e^{E^\dagger(1,2)}|0\rangle_{12}\,, \tag{A.14}$$

so that

$$\int d1_1\langle\Phi|R(1,2)\rangle = |\Phi\rangle_2\ . \tag{A.15}$$

BRST transformation. The action (A.7) is invariant under the BRST transformation

$$\delta_{\rm B}\Phi = Q_{\rm B}\Phi + g\Phi*\Phi\ . \tag{A.16}$$

The BRST operator $Q_{\rm B}$ is defined [24, 25] as

$$\begin{aligned}
Q_{\rm B} = &\frac{C_0}{2\alpha}(L_0+\tilde{L}_0-2)-i\pi_0\frac{\partial}{\partial\alpha}\\
&+\frac{i}{\alpha}\sum_{n=1}^{\infty}\left(\frac{\gamma_{-n}L_n-L_{-n}\gamma_n}{n}+\frac{\tilde{\gamma}_{-n}\tilde{L}_n-\tilde{L}_{-n}\tilde{\gamma}_n}{n}\right)\ .
\end{aligned} \tag{A.17}$$

Here L_n and $\tilde{L}_n$ ($n \in \mathbb{Z}$) are the Virasoro generators given by

$$L_n = \frac{1}{2} \sum_{m \in \mathbb{Z}} {}^\circ_\circ \alpha^N_{n+m} \alpha^M_{-m} \eta_{NM} {}^\circ_\circ \,, \quad \tilde{L}_n = \frac{1}{2} \sum_{m \in \mathbb{Z}} {}^\circ_\circ \tilde{\alpha}^N_{n+m} \tilde{\alpha}^M_{-m} \eta_{NM} {}^\circ_\circ \,, \tag{A.18}$$

where the symbol ${}^\circ_\circ \; {}^\circ_\circ$ denotes the normal ordering of the oscillators in which the non-negative modes should be placed to the right of the negative modes.

The $*$-product $\Phi * \Psi$ of two arbitrary closed string fields Φ and Ψ is given as

$$|\Phi * \Psi\rangle_4 = \int d1d2d3 \, \langle V_3(1,2,3)| \Phi\rangle_1 \, |\Psi\rangle_2 \, |R(3,4)\rangle \,, \tag{A.19}$$

where

$$\langle V_3(1,2,3)| = \delta(1,2,3) \; {}_{123}\langle 0| e^{E(1,2,3)} C(\rho_I) \mathcal{P}_{123} \frac{|\mu(1,2,3)|^2}{\alpha_1 \alpha_2 \alpha_3} \,. \tag{A.20}$$

ρ_I denotes the interaction point.

canonical quantization. Since the action (A.7) and the formulation of the OSp invariant string field theory are quite similar to those of the light-cone gauge string field theory, we can perform the canonical quantization in an analogous way. We can decompose the string field as

$$|\Phi\rangle = |\psi\rangle + |\bar{\psi}\rangle \,, \tag{A.21}$$

where $|\psi\rangle$ is the part with positive α and $|\bar{\psi}\rangle$ is the one with negative α. From the kinetic term of eq. (A.7), we can see that they satisfy the canonical commutation relation

$$\left[|\psi\rangle_r \,, |\bar{\psi}\rangle_s \right] = |R(r,s)\rangle \,. \tag{A.22}$$

From the hermiticity defined in eq. (A.12), one can deduce that $\langle\psi|$ and $\langle\bar{\psi}|$ are hermitian conjugate to $|\bar{\psi}\rangle$ and $|\psi\rangle$, respectively. We identify $|\psi\rangle$ with the annihilation mode and $|\bar{\psi}\rangle$ with the creation mode. Accordingly we define the vacuum state $|0\rangle\rangle$ in the second quantization as

$$|\psi\rangle |0\rangle\rangle = 0 \,, \quad \langle\langle 0| \langle\bar{\psi}| = 0 \,. \tag{A.23}$$

B. Overlap of three-string vertex with one boundary state

In this appendix, we evaluate the string vertex $\langle V_2(1,2); T|$ introduced in eq. (2.15) for $T = \epsilon$. $\langle V_2(1,2); T|$ can be expressed as

$$\langle V_2(1,2); T| = \langle V_2^0(1,2); T| C\left(\rho_I\right) \mathcal{P}_{12} \,, \tag{B.1}$$

where

$$\langle V_2^0(1,2); T| = \int d'3 \delta(1,2,3) \; {}_{123}\langle 0| e^{E(1,2,3)} |B_0\rangle_3^T \frac{|\mu(1,2,3)|^2}{\alpha_1 \alpha_2 \alpha_3} \,. \tag{B.2}$$

Here we present the calculations for the case $\alpha_1, \alpha_2 < 0$.

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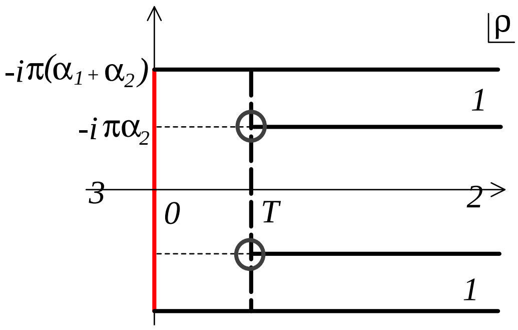

Figure 4: The ρ-plane corresponding to the string diagram depicted in figure1.

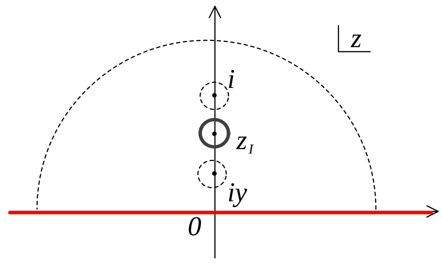

Figure 5: The upper half z-plane.

Mandelstam mapping. The vertex $\langle V_2^0(1,2);T|$ is proportional to the one that is determined by the prescription of LeClair, Peskin and Preitschopf (LPP) [26]. We refer to the latter as the LPP vertex. As we will see, we can calculate it by using the Mandelstam mapping which maps the upper half plane to the worldsheet in figure 1.

Let us introduce a complex coordinate ρ on the worldsheet so that the string diagram in figure 1 can be identified with the region depicted in figure 4 on the ρ-plane. Each portion of the ρ-plane corresponding to the r-th external string ($r = 1, 2$) is identified with the unit disk $|w_r| \leq 1$ of string r by the relation

$$\begin{aligned} &\rho = \alpha_r \zeta_r + T + i\beta_r\,, \quad \beta_r = -\alpha_2\pi - \alpha_r \sigma_I^{(r)}\,, \\ &\zeta_r (= \tau_r + i\sigma_r) = \ln w_r\,, \qquad \tau_r \leq 0\,, \quad -\pi \leq \sigma_r \leq \pi\ . \end{aligned} \tag{B.3}$$

Here $\rho_I = T - i\pi\alpha_2$ is the interaction point on the ρ-plane and $\sigma_I^{(r)}$ is the value of the σ_r coordinate where the r-th string interacts. We set $\sigma_I^{(1)} = \pi$ and $\sigma_I^{(2)} = -\pi$. Therefore we have

$$\beta_1 = -(\alpha_1 + \alpha_2)\pi\,, \quad \beta_2 = 0\ . \tag{B.4}$$

The string diagram described by figure 1 has one hole and two punctures at infinity corresponding to the two external strings, strings 1 and 2. Since the topology of this diagram is a disk with two punctures, the ρ-plane (figure 4) can be mapped to the complex upper half z-plane (figure 5) with two punctures. These two surfaces are related by the Mandelstam mapping

$$\rho(z) = \alpha_1 \ln \frac{z - Z_1}{z - \bar{Z}_1} + \alpha_2 \ln \frac{z - Z_2}{z - \bar{Z}_2}\,, \tag{B.5}$$

where the point $z = Z_r$ ($r = 1, 2$) is the puncture corresponding to the origin of the unit disk $|w_r| < 1$ of string r. We can set $Z_1 = iy$ and $Z_2 = i$, where y is a real parameter with $0 < y < 1$. The interaction point z_I on the z-plane is determined by $\frac{d\rho}{dz}(z_I) = 0$. This yields

$$z_I = i\sqrt{\frac{(\alpha_1 + \alpha_2 y)y}{\alpha_1 y + \alpha_2}}\ . \tag{B.6}$$

Here we have used $\alpha_1, \alpha_2 < 0$, $0 < y < 1$ and $\mathrm{Im}\, z_I > 0$. Eq. (B.6) leads to

$$T = \mathrm{Re}\, \rho(z_I) = \alpha_1 \ln \left| \frac{z_I - iy}{z_I + iy} \right| + \alpha_2 \ln \left| \frac{z_I - i}{z_I + i} \right| \ . \tag{B.7}$$

From this relation, we find that in the small T limit, $T = \epsilon \ll 1$, we have

$$y \simeq \frac{1}{16\alpha_1\alpha_2}\epsilon^2 + \mathcal{O}(\epsilon^4) \ . \tag{B.8}$$

For later use, we consider the limit $T \to \infty$ as well. In this limit, $y \sim 1$. In fact,

$$T \simeq \hat{\tau}_0 - (\alpha_1 + \alpha_2) \ln 2 + (\alpha_1 + \alpha_2) \ln(1 - y) + \mathcal{O}(1 - y) \, , \tag{B.9}$$

where

$$\hat{\tau}_0 = \alpha_1 \ln |\alpha_1| + \alpha_2 \ln |\alpha_2| - (\alpha_1 + \alpha_2) \ln |\alpha_1 + \alpha_2| \ . \tag{B.10}$$

Neumann coefficients. The real axis of the z-plane corresponds to the worldsheet boundary attached to $|B_0\rangle_3$. Because of the boundary conditions (2.1) satisfied by the worldsheet variables $X^N = (X^\mu, X^i, C, \bar{C})$ on the boundary state $|B_0\rangle$, the two-point functions of $X^N(z, \bar{z})$ on the z-plane become

$$G^{NM}_{\mathrm{UHP}}(z, \bar{z}; z', \bar{z}') = \langle X^N(z, \bar{z}) X^M(z', \bar{z}') \rangle = -\eta^{NM} \ln |z - z'|^2 - D^{NM} \ln |z - \bar{z}'|^2 \, , \tag{B.11}$$

where D^{NM} is the tensor introduced in eq. (2.3).

The vertex $\langle V^0_2(1,2); T|$ introduced in eq. (B.2) takes the form

$$\langle V^0_2(1,2); T| = 2\delta(\alpha_1 + \alpha_2 + \alpha_3)(2\pi)^{p+1} \delta^{p+1}_{\mathrm{N}}(p_1 + p_2)\, \mathcal{K}_2(1,2;T)\, \langle V^0_{2,\mathrm{LPP}}(1,2); T| \, , \tag{B.12}$$

where $\langle V^0_{2,\mathrm{LPP}}(1,2); T|$ is the LPP vertex, and the factor $\mathcal{K}_2(1,2;T)$ depends only on the zero-modes and the moduli. The LPP vertex has the structure

$$\begin{aligned} &\langle V^0_{2,\mathrm{LPP}}(1,2); T| \\ &\quad = {}_{12}\langle 0| \exp \Bigg[\sum_{n,m=0}^{\infty} \sum_{r,s=1,2} \Big\{ \frac{1}{2} \Big(\bar{N}^{(2)rs}_{nm}\, \alpha^{N(r)}_n\, \alpha^{M(s)}_m + \bar{N}^{(2)\tilde{r}\tilde{s}}_{nm}\, \tilde{\alpha}^{N(r)}_n \tilde{\alpha}^{M(s)}_m \Big) \eta_{NM} \\ &\qquad\qquad + \frac{1}{2} \Big(\bar{N}^{(2)r\tilde{s}}_{nm}\, \alpha^{N(r)}_n \tilde{\alpha}^{M(s)}_m + \bar{N}^{(2)\tilde{r}s}_{nm}\, \tilde{\alpha}^{N(r)}_n \alpha^{M(s)}_m \Big) D_{NM} \Big\} \Bigg] \, , \end{aligned} \tag{B.13}$$

for Wick's theorem to hold. The Neumann coefficients $\bar{N}^{(2)rs}_{nm}$, $\bar{N}^{(2)\tilde{r}\tilde{s}}_{nm}$ and $\bar{N}^{(2)r\tilde{s}}_{nm}$ are determined by requiring that the following equation should hold [26],

$$\begin{aligned} &\int d'1 d'2\, \langle V^0_{2,\mathrm{LPP}}(1,2); T| X^{N(r)}(w_r, \bar{w}_r) X^{M(s)}(w'_s, \bar{w}'_s) |0\rangle_{12} \\ &\qquad \times \prod_{r'=1,2} (2\pi)^{26} \delta^{26}(p_{r'}) i \bar{\pi}^{(r')}_0 \pi^{(r')}_0 = G^{NM}_{\mathrm{UHP}}(z_r, \bar{z}_r; z'_s, \bar{z}'_s) \, , \end{aligned} \tag{B.14}$$

where z_r and z'_s are the points on the z-plane corresponding to the points w_r and w'_s on the unit disks of strings r and s, respectively.

Using eq. (B.14), one can show that the Neumann coefficients are given as

$$
\begin{aligned}
\bar{N}^{(2)rs}_{nm} &= \left(\bar{N}^{(2)\tilde{r}\tilde{s}}_{nm}\right)^* = \frac{1}{nm}\oint_{Z_r}\frac{dz}{2\pi i}\oint_{Z_s}\frac{dz'}{2\pi i}\frac{e^{-n\zeta_r(z)-m\zeta'_s(z')}}{(z-z')^2}\,,\\
\bar{N}^{(2)r\tilde{s}}_{nm} &= \left(\bar{N}^{(2)\tilde{r}s}_{nm}\right)^* = \frac{1}{nm}\oint_{Z_r}\frac{dz}{2\pi i}\oint_{\bar{Z}_s}\frac{d\bar{z}'}{2\pi i}\frac{e^{-n\zeta_r(z)-m\bar{\zeta}'_s(\bar{z}')}}{(z-\bar{z}')^2}\,,\\
\bar{N}^{(2)rs}_{n0} &= \left(\bar{N}^{(2)\tilde{r}\tilde{s}}_{n0}\right)^* = \frac{1}{n}\oint_{Z_r}\frac{dz}{2\pi i}\frac{e^{-n\zeta_r(z)}}{z-Z_s}\,,\\
\bar{N}^{(2)r\tilde{s}}_{n0} &= \left(\bar{N}^{(2)\tilde{r}s}_{n0}\right)^* = \frac{1}{n}\oint_{Z_r}\frac{dz}{2\pi i}\frac{e^{-n\zeta_r(z)}}{z-\bar{Z}_s}\,,\\
\bar{N}^{(2)rs}_{00} &= \left(\bar{N}^{(2)\tilde{r}\tilde{s}}_{00}\right)^* = \ln(Z_r-Z_s)\quad (r\neq s)\,,\\
\bar{N}^{(2)r\tilde{s}}_{00} &= \left(\bar{N}^{(2)\tilde{r}s}_{00}\right)^* = \ln(Z_r-\bar{Z}_s)\quad (r\neq s)\,,\\
\bar{N}^{(2)rr}_{00} &= \left(\bar{N}^{(2)\tilde{r}\tilde{r}}_{00}\right)^* = \ln(Z_r-\bar{Z}_r)-\sum_{s\neq r}\frac{\alpha_s}{\alpha_r}\left\{\ln(Z_r-Z_s)-\ln(Z_r-\bar{Z}_s)\right\}+\frac{T+i\beta_r}{\alpha_r}\,,\\
\bar{N}^{(2)r\tilde{r}}_{00} &= \left(\bar{N}^{(2)\tilde{r}r}_{00}\right)^* = \ln(Z_r-\bar{Z}_r)\,,
\end{aligned}
\tag{B.15}
$$

for $n,m\geq 1$. Here we have used the convention for the orientation of the $\bar{z}$ integration such that $\oint_0 \frac{d\bar{z}}{2\pi i}\frac{1}{\bar{z}}=1$.

$\mathcal{K}_2(1,2;T)$. The central charge of the worldsheet CFT of the OSp invariant string theory is 24 and not 0. Therefore the Generalized Gluing and Resmoothing Theorem [27] does not hold in this case and thus $\mathcal{K}_2(1,2;T)\neq 1$. Since the three-string vertex $\langle V_3^0(1,2,3)|$ is defined assuming that the ρ-plane is endowed with the metric

$$ds^2 = d\rho d\bar{\rho}\,, \tag{B.16}$$

the oscillator independent part $\mathcal{K}_2(1,2;T)$ is the partition function of the CFT on the ρ-plane (figure 4) with the metric given in eq. (B.16). As explained in [28], its dependence on α_1, α_2 and the moduli T can be determined through CFT technique by evaluating the Liouville action associated with the conformal mapping (B.5) between the ρ-plane and the upper half z-plane with small circles around z_I, Z_r ($r=1,2$) and ∞ excised. Collecting the contributions from these holes, we obtain

$$\mathcal{K}_2(1,2;T)\propto\left|\lim_{z\to\infty}\left(z^2\frac{d\rho(z)}{dz}\right)\right|^2\prod_{r=1,2}\left(|\alpha_r|^{-2}\left|\frac{dw_r}{dz}(Z_r)\right|^2\right)\left|\frac{d^2\rho}{dz^2}(z_I)\right|^{-1}\,. \tag{B.17}$$

Therefore we can see that $\mathcal{K}_2(1,2;T)$ is expressed as

$$\mathcal{K}_2(1,2;T)=\mathcal{K}_0\frac{1}{\alpha_1\alpha_2}\sqrt{\frac{(\alpha_1 y+\alpha_2)y}{\alpha_1+\alpha_2 y}}\,\frac{(1-y^2)^2}{(\alpha_1 y+\alpha_2)16y^2}\,e^{-2\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}\right)T-2\left(\frac{\alpha_2}{\alpha_1}+\frac{\alpha_1}{\alpha_2}\right)\ln\frac{1+y}{1-y}}\,, \tag{B.18}$$

where $\mathcal{K}_0$ is a constant independent of α_1, α_2 and T. $\mathcal{K}_0$ can be determined by comparing the left and right hand sides of the equation

$$\begin{aligned}\int d'1d'2d'3\,\langle V_3^0(1,2,3)|B_0\rangle_3^T|0\rangle_{12}\prod_{r=1,2}(2\pi)^{26}\delta^{26}(p_r)i\bar{\pi}_0^{(r)}\pi_0^{(r)}\\ =\int d'1d'2\;2\delta(\alpha_1+\alpha_2+\alpha_3)(2\pi)^{p+1}\delta_{\mathrm{N}}^{p+1}(p_1+p_2)\,\mathcal{K}_2(1,2;T)\\ \times\langle V_{2,\mathrm{LPP}}^0(1,2);T|0\rangle_{12}\prod_{r=1,2}(2\pi)^{26}\delta^{26}(p_r)i\bar{\pi}_0^{(r)}\pi_0^{(r)}\,,\end{aligned}\tag{B.19}$$

in the $T\to\infty$ limit. One can readily evaluate the left hand side of the above equation because the non-zero oscillation modes do not contribute in this limit. Using eq. (B.9), we find that $\mathcal{K}_0=-1$.

$C(\rho_I)$. The effect of inserting $C(\rho_I)$ can be described as follows:

$$\begin{aligned}\langle V_{2,\mathrm{LPP}}(1,2);T| &\equiv \langle V_{2,\mathrm{LPP}}^0(1,2);T|C(\rho_I)\\ &=\langle V_{2,\mathrm{LPP}}^0(1,2);T|\left[\sum_{n=0}^{\infty}\sum_{r=1,2}\left(M_{\mathrm{UHP}}{}_n^r i\gamma_n^{(r)}+M_{\mathrm{UHP}}{}_n^{\tilde{r}}i\tilde{\gamma}_n^{(r)}\right)\right].\end{aligned}\tag{B.20}$$

The coefficients $M_{\mathrm{UHP}}{}_n^r$ and $M_{\mathrm{UHP}}{}_n^{\tilde{r}}$ can be determined by the LPP prescription, i.e. we require that

$$\begin{aligned}\int d'1d'2\langle V_{2,\mathrm{LPP}}(1,2);T|\bar{C}^{(r)}(w_r,\bar{w}_r)|0\rangle_{12}\prod_{r=1,2}(2\pi)^{26}\delta^{26}(p_r)i\bar{\pi}_0^{(r)}\pi_0^{(r)}\\ =G_{\mathrm{UHP}}^{C\bar{C}}(z_I,\bar{z}_I;z_r,\bar{z}_r)=i\Big[\ln(z_I-z_r)+\ln(\bar{z}_I-\bar{z}_r)-\ln(z_I-\bar{z}_r)-\ln(\bar{z}_I-z_r)\Big].\end{aligned}\tag{B.21}$$

This yields

$$M_{\mathrm{UHP}}{}_n^r=(M_{\mathrm{UHP}}{}_n^{\tilde{r}})^*=-\frac{i}{n}\oint_{Z_r}\frac{dz_r}{2\pi i}\,e^{-n\zeta_r(z_r)}\left(\frac{i}{z_r-z_I}-\frac{i}{z_r-\bar{z}_I}\right)\tag{B.22}$$

for $n\geq 1$ and

$$M_{\mathrm{UHP}}{}_0^r+M_{\mathrm{UHP}}{}_0^{\tilde{r}}=\ln(z_I-Z_r)+\ln(\bar{z}_I-\bar{Z}_r)-\ln(z_I-\bar{Z}_r)-\ln(\bar{z}_I-Z_r)\,.\tag{B.23}$$

$\langle V_2(1,2);\epsilon|$. Collecting the results obtained in the above, we eventually get the vertex $\langle V_2(1,2);T|$. Now that we obtain the complete expression of the vertex $\langle V_2(1,2);T|$, let us consider the $T=\epsilon\to 0$ limit. It is intuitively obvious that $\langle V_{2,\mathrm{LPP}}^0(1,2);\epsilon|$ is proportional to a product of boundary states in this limit. It is straightforward to show that

$$(2\pi)^{p+1}\delta_{\mathrm{N}}^{p+1}(p_1+p_2)\langle V_{2,\mathrm{LPP}}^0(1,2);\epsilon|\sim\frac{1}{(16\pi)^{\frac{p+1}{2}}(-\ln\epsilon)^{\frac{p+1}{2}}}\,{}_1^{\epsilon}\langle B_0|\,{}_2^{\epsilon}\langle B_0|\,,\tag{B.24}$$

in the leading order. Therefore, in evaluating $\langle V_2(1,2);\epsilon|=\langle V_2^0(1,2);\epsilon|C\left(\rho_I\right)\mathcal{P}_{12}$, only the term proportional to $\pi_0^{(r)}$ from $C(\rho_I)$ survives the level matching condition and we obtain

$$
\begin{aligned}
&(2\pi)^{p+1}\delta_{\mathrm{N}}^{p+1}(p_1+p_2)\langle V_{2,\mathrm{LPP}}(1,2);\epsilon|\mathcal{P}_{12}\\
&\sim \frac{1}{(16\pi)^{\frac{p+1}{2}}(-\ln\epsilon)^{\frac{p+1}{2}}}\,{}_1^{\epsilon}\langle B_0|\,{}_2^{\epsilon}\langle B_0|\left(\frac{i}{\alpha_1}\pi_0^{(1)}+\frac{i}{\alpha_2}\pi_0^{(2)}\right)\mathcal{P}_{12}\ .
\end{aligned}
\tag{B.25}
$$

Evaluating $\mathcal{K}_2(1,2;\epsilon)$, we finally obtain

$$
\begin{aligned}
\langle V_2(1,2);\epsilon| \sim 2\delta(\alpha_1+\alpha_2+\alpha_3)&\frac{1}{(16\pi)^{\frac{p+1}{2}}}\frac{4}{\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}\\
&\times{}_1^{\epsilon}\langle B_0|\,{}_2^{\epsilon}\langle B_0|\left(\frac{i}{\alpha_1}\pi_0^{(1)}+\frac{i}{\alpha_2}\pi_0^{(2)}\right)\mathcal{P}_{12}\ .
\end{aligned}
\tag{B.26}
$$

The case $\alpha_1, \alpha_2 > 0$ can be treated in the same way and one can show that eq. (B.26) holds also in this case.

C. Overlap of three-string vertex with two boundary states

In this appendix, we investigate the vertex $\langle V_1(3);\epsilon|$ introduced in eq. (2.16). The calculation proceeds in the same way as the one above. We begin by expressing $\langle V_1(3);T|$ as

$$
\langle V_1(3);T| = \langle V_1^0(3);T|C(\rho_I)\mathcal{P}_3\,,
\tag{C.1}
$$

where

$$
\langle V_1^0(3);T| \equiv \int d'1d'2\,\delta(1,2,3)\ {}_{123}\langle 0|e^{E(1,2,3)}|B_0\rangle_1^T|B_0\rangle_2^T\frac{|\mu(1,2,3)|^2}{\alpha_1\alpha_2\alpha_3}\ .
\tag{C.2}
$$

Here we present the calculations for the case $\alpha_1, \alpha_2 > 0$.

Mandelstam mapping. The complex ρ-plane indicating the string diagram figure 2 is described by figure 6. The region of the ρ-plane corresponding to the external string, string 3, is identified with the unit disk $|w_3| \le 1$ of this string through the relation

$$
\begin{aligned}
&\rho = \alpha_3\zeta_3 + T + i\beta_3\,, \quad \beta_3 = \alpha_1\pi - \alpha_3\sigma_I^{(3)}\,,\\
&\zeta_3(=\tau_3+i\sigma_3) = \ln w_3\,, \qquad \tau_3 \le 0\,, \quad -\pi \le \sigma_3 \le \pi\ .
\end{aligned}
\tag{C.3}
$$

Here $\rho_I = T + i\pi\alpha_1$ (and $\bar\rho_I$) is the interaction point on the ρ-plane and $\sigma_I^{(3)}$ denotes the value of the σ_3 coordinate of the interaction point of string 3. We set $\sigma_I^{(3)} = \pi\alpha_1/\alpha_3$ so that

$$
\beta_3 = 0\ .
\tag{C.4}
$$

The topology of the string diagram figure 2 is an annulus with a puncture corresponding to string 3. Therefore the ρ-plane can be mapped to a rectangle with a puncture on the complex ν-plane (figure 7). We take this rectangle to be the region defined by $-\frac{1}{2} \le \mathrm{Re}\,\nu \le 0$ and $-\frac{\tau}{2} \le \mathrm{Im}\,\nu \le \frac{\tau}{2}$. Here τ ($\tau \in i\mathbb{R}$) is the moduli parameter and the identification

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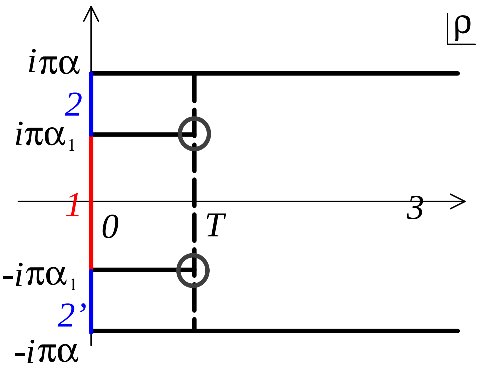

Figure 6: The ρ-plane corresponding to the string diagram depicted by figure2.

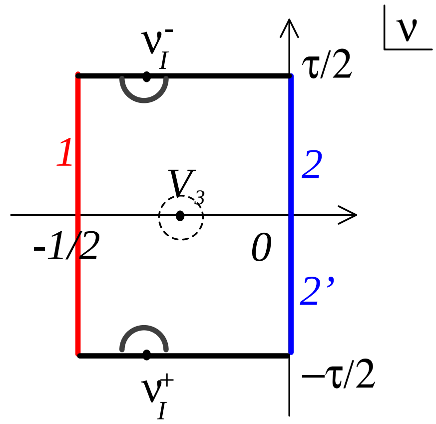

Figure 7: The rectangle on the ν-plane.

$\nu \cong \nu + \tau$ should be made. These two surfaces are related by the Mandelstam mapping[3]

$$\rho(\nu) = \alpha \ln \frac{\vartheta_1(\nu + V_3|\tau)}{\vartheta_1(\nu - V_3|\tau)}\,, \tag{C.5}$$

where $\alpha = \alpha_1 + \alpha_2 = -\alpha_3 > 0$, $V_3 = -\frac{\alpha_1}{2\alpha}$ and $\vartheta_i(\nu|\tau)$ $(i = 1, \ldots, 4)$ are the theta functions. The point $\nu = V_3$ is the puncture corresponding to the origin $w_3 = 0$ of the unit disk $|w_3| \leq 1$ of string 3. We may parametrize the interaction points ν_I^- and ν_I^+ on the ν-plane corresponding to ρ_I and $\bar{\rho}_I$ on the ρ-plane as $\nu_I^{\pm} = -y \mp \frac{\tau}{2}$ with $y \in \mathbb{R}$, $0 \leq y \leq \frac{1}{2}$. These are determined by $\frac{d\rho}{d\nu}(\nu_I^{\pm}) = 0$. This yields

$$g_4\left(\frac{\alpha_1}{2\alpha} + y \,\Big|\, \tau\right) + g_4\left(\frac{\alpha_1}{2\alpha} - y \,\Big|\, \tau\right) = 0\,, \tag{C.6}$$

where $g_i(\nu|\tau) = \partial_\nu \ln \vartheta_i(\nu|\tau)$. The relation $\mathrm{Re}\,\rho(\nu_I^{\pm}) = T$ leads to

$$T = \alpha \ln \frac{\vartheta_4\left(\frac{\alpha_1}{2\alpha} + y \,\middle|\, \tau\right)}{\vartheta_4\left(\frac{\alpha_1}{2\alpha} - y \,\middle|\, \tau\right)}\,. \tag{C.7}$$

It follows from eqs. (C.6) and (C.7) that in the small T limit, $T = \epsilon \ll 1$, the parameters τ and y behave as follows [29, 31]:

$$q^{\frac{1}{2}} \equiv e^{i\pi\tau} \simeq \frac{\epsilon}{4\alpha \sin\left(\pi\frac{\alpha_1}{\alpha}\right)} + \mathcal{O}(\epsilon^3)\,, \quad y \simeq \frac{1}{4} - \frac{1}{\pi}\cos\left(\pi\frac{\alpha_1}{\alpha}\right) q^{\frac{1}{2}} + \mathcal{O}(q^{\frac{3}{2}})\,. \tag{C.8}$$

Therefore we find that in this limit the moduli parameter $-i\tau$ becomes infinity. For later use, we consider the behavior of τ and y in the $T \to \infty$ limit as well. In this limit, the moduli parameter τ tends to 0. In fact, we have

$$T \sim \frac{\alpha_1\alpha_2\pi}{\alpha}\frac{i}{\tau} + \hat{\tau}_0\,, \quad y \sim \frac{\alpha_1}{2\alpha} + \frac{i}{2\pi}\tau \ln\frac{\alpha_1}{\alpha_2}\,. \tag{C.9}$$

[3] The Mandelstam mapping (C.5) is essentially the same as the one in [29]. The rectangle on the ν-plane introduced here is the dual annulus of the rectangle on the u-plane considered in [29]. These are related by $\nu = \frac{u}{\tilde{\tau}}$, where $\tilde{\tau} = -\frac{1}{\tau}$. See also [30 – 32].

Neumann coefficients. The part $-\pi\alpha_1 \leq \operatorname{Im}\rho \leq \pi\alpha_1$ of the boundary $\operatorname{Re}\rho = 0$ of the ρ-plane where the ρ-plane is attached to $|B_0\rangle_1$ corresponds to the side $\operatorname{Re}\nu = -\frac{1}{2}$ of the rectangle on the ν-plane. The remaining part of the boundary of the ρ-plane where the ρ-plane is attached to $|B_0\rangle_2$ corresponds to the the other side $\operatorname{Re}\nu = 0$ of the rectangle on the ν-plane. Therefore, on the ν-plane the worldsheet variables $X^N(\nu,\bar{\nu})$ satisfy the Neumann and the Dirichlet boundary conditions according to eq. (2.1) on the two sides, $\operatorname{Re}\nu = -\frac{1}{2}$ and $\operatorname{Re}\nu = 0$, of the rectangle and the periodic boundary condition $X^N(\nu+\tau,\bar{\nu}-\tau) = X^N(\nu,\bar{\nu})$ along the imaginary axis. It follows that the two-point functions of $X^N(\nu,\bar{\nu})$ on the ν-plane become

$$\begin{aligned} G^{NM}_{\text{rectan.}}(\nu,\bar{\nu};\nu',\bar{\nu}') &= \langle X^N(\nu,\bar{\nu})X^M(\nu',\bar{\nu}')\rangle \\ &= -\eta^{NM}\ln\vartheta_1\left(\nu-\nu'\,\middle|\,\tau\right) - \eta^{NM}\ln\vartheta_1\left(\bar{\nu}-\bar{\nu}'\,\middle|\,\tau\right) \\ &\quad - D^{NM}\ln\vartheta_1\left(\nu+\bar{\nu}'\,\middle|\,\tau\right) - D^{NM}\ln\vartheta_1\left(\bar{\nu}+\nu'\,\middle|\,\tau\right) + f^{NM}(\nu,\bar{\nu};\nu',\bar{\nu}')\,, \end{aligned} \tag{C.10}$$

where $f^{NM}(\nu,\bar{\nu};\nu',\bar{\nu}')$ are the terms necessary for the periodicity of the two-point functions $G^{NM}_{\text{rectan.}}(\nu,\bar{\nu};\nu',\bar{\nu}')$ along the imaginary axis of the ν-plane, defined as

$$f^{\mu\lambda}(\nu,\bar{\nu};\nu',\bar{\nu}') = -\eta^{\mu\lambda}\frac{\pi i}{\tau}\left(\nu-\nu'-\bar{\nu}+\bar{\nu}'\right)^2 \tag{C.11}$$

for $\mu,\lambda \in \mathrm{N}$, and 0 otherwise. In eq. (C.10), we have used the relation

$$\overline{\vartheta_1(\nu|\tau)} = \vartheta_1(\bar{\nu}|\tau) \tag{C.12}$$

for $\tau \in i\mathbb{R}$.

The vertex $\langle V^0_1(3);T|$ can be expressed as

$$\langle V^0_1(3);T| = 2\delta(\alpha_1+\alpha_2+\alpha_3)(2\pi)^{p+1}\delta^{p+1}_{\mathrm{N}}(p_3)\mathcal{K}_1(3;T)\langle V^0_{1,\text{LPP}}(3);T|\,, \tag{C.13}$$

where $\langle V^0_{1,\text{LPP}}(3);T|$ is the LPP vertex, which is of the form

$$\begin{aligned} &\langle V^0_{1,\text{LPP}}(3);T| \\ &= {}_3\langle 0|\exp\left[\sum_{n,m=0}^{\infty}\left\{\frac{1}{2}\left(\bar{N}^{hh}_{nm,NM}\,\alpha^{N(3)}_n\,\alpha^{M(3)}_m + \bar{N}^{aa}_{nm,NM}\,\tilde{\alpha}^{N(3)}_n\tilde{\alpha}^{M(3)}_m\right.\right.\right. \\ &\qquad\qquad \left.\left.\left. +\bar{N}^{ha}_{nm,NM}\,\alpha^{N(3)}_n\tilde{\alpha}^{M(3)}_m + \bar{N}^{ah}_{nm,NM}\,\tilde{\alpha}^{N(3)}_n\alpha^{M(3)}_m\right)\right\}\right]\,, \end{aligned} \tag{C.14}$$

and the remaining factor $\mathcal{K}_1(3;T)$ is independent of the non-zero oscillation modes.

The Neumann coefficients $\bar{N}^{hh}_{nm,NM}$, $\bar{N}^{aa}_{nm,NM}$, $\bar{N}^{ha}_{nm,NM}$, $\bar{N}^{ah}_{nm,NM}$ are determined by the equation

$$\begin{aligned} &\int d'3\langle V^0_{1,\text{LPP}}(3);T|X^{N(3)}(w_3,\bar{w}_3)X^{M(3)}(w'_3,\bar{w}'_3)|0\rangle_3(2\pi)^{26}\delta^{26}(p_3)i\bar{\pi}^{(3)}_0\pi^{(3)}_0 \\ &\qquad = G^{NM}_{\text{rectan.}}(\nu_3,\bar{\nu}_3;\nu'_3,\bar{\nu}'_3)\,, \end{aligned} \tag{C.15}$$

where ν_3 and ν_3' are the points on the ν-plane corresponding to the points w_3 and w_3' on the unit disk of string 3 respectively. We obtain

$$\begin{aligned}
\bar{N}^{hh,NM}_{nm} &= \left(\bar{N}^{aa,NM}_{nm}\right)^* = \frac{-1}{nm}\oint_{V_3}\frac{d\nu}{2\pi i}\oint_{V_3}\frac{d\nu'}{2\pi i}e^{-n\zeta_3(\nu)-m\zeta_3'(\nu')}\partial_\nu\partial_{\nu'}G^{NM}_{\rm rectan.}(\nu,\bar{\nu};\nu',\bar{\nu}')\,,\\
\bar{N}^{ha,NM}_{nm} &= \left(\bar{N}^{ah,NM}_{nm}\right)^* = \frac{-1}{nm}\oint_{V_3}\frac{d\nu}{2\pi i}\oint_{V_3}\frac{d\bar{\nu}'}{2\pi i}e^{-n\zeta_3(\nu)-m\bar{\zeta}_3'(\bar{\nu}')}\partial_\nu\partial_{\bar{\nu}'}G^{NM}_{\rm rectan.}(\nu,\bar{\nu};\nu',\bar{\nu}')\,,\\
\frac{1}{2}\left(\bar{N}^{hh,NM}_{n0}+\bar{N}^{ha,NM}_{n0}\right) &= \frac{1}{2}\left(\bar{N}^{aa,NM}_{n0}+\bar{N}^{ah,NM}_{n0}\right)^*\\
&= -\frac{1}{2n}\oint_{V_3}\frac{d\nu}{2\pi i}e^{-n\zeta_3(\nu)}\partial_\nu G^{NM}_{\rm rectan.}(\nu,\bar{\nu};V_3,V_3)\,,\\
\frac{1}{4}\left(\bar{N}^{hh,NM}_{00}+\bar{N}^{ha,NM}_{00}+\bar{N}^{ah,NM}_{00}+\bar{N}^{aa,NM}_{00}\right)&\\
&= -\frac{1}{4}\lim_{\nu\to V_3}\left(G^{NM}_{\rm rectan.}(\nu,\bar{\nu};V_3,V_3)+\zeta_3(\nu)+\bar{\zeta}_3(\bar{\nu})\right)\,. \qquad \text{(C.16)}
\end{aligned}$$

$\mathcal{K}_1(3;T)$. The prefactor $\mathcal{K}_1(3;T)$ can be determined through the method in [28] again. We excise small semi-circles around the interaction points $\nu=\nu_I^\pm$ and a small circle around the puncture $\nu=V_3$ on the ν-plane. This time, besides the contributions from these holes, we should include the moduli dependence of the partition function, and we find

$$\mathcal{K}_1(3;T)\propto\left(|\alpha|^{-2}\left|\frac{dw_3}{d\nu}(V_3)\right|^2\right)|c_I|^{-1}|\tau|^{-\frac{p+1}{2}}\eta(\tau)^{-24}, \qquad \text{(C.17)}$$

where $\eta(\tau)$ is the Dedekind eta function and c_I is defined by

$$c_I=\frac{d^2\rho}{d\nu^2}(\nu_I^-)\,. \qquad \text{(C.18)}$$

Thus we obtain

$$\mathcal{K}_1(3;T)=\mathcal{K}_0'\frac{(2\pi)^2e^{\frac{2T}{\alpha}}}{(-i\tau)^{\frac{p+1}{2}}\eta(\tau)^{18}\alpha^2c_I\vartheta_1\left(\frac{\alpha_1}{\alpha}\,\middle|\,\tau\right)^2}\,, \qquad \text{(C.19)}$$

where $\mathcal{K}_0'$ is a numerical factor which cannot be determined by this method. The factor $\mathcal{K}_0'$ can be fixed by comparing the behaviors in the $T\to\infty$ limit of the left and right hand sides of the following equation,

$$\begin{aligned}
&\int d'1d'2d'3\langle V_3^0(1,2,3)|B_0\rangle_1^T|B_0\rangle_2^T|0\rangle_3(2\pi)^{26}\delta^{26}(p_3)i\bar{\pi}_0^{(3)}\pi_0^{(3)}\\
&\quad=\int d'3\;2\delta(\alpha_1+\alpha_2+\alpha_3)(2\pi)^{p+1}\delta_{\rm N}^{p+1}(p_3)\mathcal{K}_1(3;T)\\
&\qquad\times\langle V^0_{1,\rm LPP}(3);T|0\rangle_3(2\pi)^{26}\delta^{26}(p_3)i\bar{\pi}_0^{(3)}\pi_0^{(3)}\,. \qquad \text{(C.20)}
\end{aligned}$$

By making use of eq. (C.9), we find that

$$\mathcal{K}_0'=\frac{(2\pi)^{p+1}}{(2\pi)^{25}}\,. \qquad \text{(C.21)}$$

$C(\rho_I)$. Let us consider the effect of the insertion of the ghost field C at the interaction point. In the same way as eq. (B.20), this can be described by

$$\begin{aligned}\langle V_{1,\mathrm{LPP}}(3);T| &\equiv \langle V^0_{1,\mathrm{LPP}}(3);T|C(\rho_I)\\ &= \langle V^0_{1,\mathrm{LPP}}(3);T|\sum_{n=0}^{\infty}\left(M_{\mathrm{rectan}.n}{}^{h}\, i\gamma_n^{(3)} + M_{\mathrm{rectan}.n}{}^{a}\, i\tilde{\gamma}_n^{(3)}\right)\ .\end{aligned} \tag{C.22}$$

The coefficients $M_{\mathrm{rectan}.n}{}^{h}$ and $M_{\mathrm{rectan}.n}{}^{a}$ can be determined through the LPP prescription by requiring that

$$\begin{aligned}&\int d'3\langle V_{1,\mathrm{LPP}}(3);T|\bar{C}^{(3)}(w_3,\bar{w}_3)|0\rangle_3(2\pi)^{26}\delta^{26}(p_3)i\bar{\pi}_0^{(3)}\pi_0^{(3)}\\ &= G^{C\bar{C}}_{\mathrm{rectan.}}(\nu_I^-,\bar{\nu}_I^-;\nu_3,\bar{\nu}_3)\\ &= i\Big[\ln\vartheta_1(\nu_I^- - \nu_3|\tau) + \ln\vartheta_1(\bar{\nu}_I^- - \bar{\nu}_3|\tau) - \ln\vartheta_1(\nu_I^- + \bar{\nu}_3|\tau) - \ln\vartheta_1(\bar{\nu}_I^- + \nu_3|\tau)\Big].\end{aligned} \tag{C.23}$$

It follows that the coefficient of the zero mode $\gamma_0^{(3)} = \tilde{\gamma}_0^{(3)} = \pi_0^{(3)}$ is

$$\begin{aligned}&M_{\mathrm{rectan}.0}{}^{h} + M_{\mathrm{rectan}.0}{}^{a}\\ &\quad= \ln\vartheta_1(\nu_I^- - V_3|\tau) + \ln\vartheta_1(\bar{\nu}_I^- - V_3|\tau) - \ln\vartheta_1(\nu_I^- + V_3|\tau) - \ln\vartheta_1(\bar{\nu}_I^- + V_3|\tau)\\ &\quad= 2\ln\frac{\vartheta_4\left(\frac{\alpha_1}{2\alpha} - y\,\middle|\,\tau\right)}{\vartheta_4\left(\frac{\alpha_1}{2\alpha} + y\,\middle|\,\tau\right)}\,,\end{aligned} \tag{C.24}$$

and those of the non-zero modes are

$$M_{\mathrm{rectan}.n}{}^{h} = (M_{\mathrm{rectan}.n}{}^{a})^* = -\frac{1}{n}\oint_{\nu=V_3}\frac{d\nu}{2\pi i}\Big[g_1(\nu_I^- - \nu|\tau) + g_1(\bar{\nu}_I^- + \nu|\tau)\Big] \tag{C.25}$$

for $n \geq 1$.

$\langle V_1(3);\epsilon|$. Collecting all the results obtained in the above, we eventually get the complete expression of the vertex $\langle V_1(3);T|$. Let us take $T = \epsilon \to 0$ limit. Again it is intuitively obvious and straightforward to show that

$$(2\pi)^{p+1}\delta_{\mathrm{N}}^{p+1}(p_3)\langle V^0_{1,\mathrm{LPP}}(3);\epsilon| \sim {}^{\epsilon}_{3}\langle B_0|\,, \tag{C.26}$$

in the leading order. It follows that only the $\pi_0^{(3)}$ from $C(\rho_I)$ survives the level matching projection and thus

$$(2\pi)^{p+1}\delta_{\mathrm{N}}^{p+1}(p_3)\langle V_{1,\mathrm{LPP}}(3);\epsilon|\mathcal{P}_3 \sim {}^{\epsilon}_{3}\langle B_0|\frac{2i}{\alpha_3}\pi_0^{(3)}\mathcal{P}_3\ . \tag{C.27}$$

Evaluating $\mathcal{K}_1(3,\epsilon)$, we eventually get

$$\langle V_1(3);\epsilon| \sim -2\delta(\alpha_1+\alpha_2+\alpha_3)\frac{(4\pi^3)^{\frac{p+1}{2}}}{(2\pi)^{25}}\frac{4}{\epsilon^2(-\ln\epsilon)^{\frac{p+1}{2}}}{}^{\epsilon}_{3}\langle B_0|\frac{2i}{\alpha_3}\pi_0^{(3)}\mathcal{P}_3\ . \tag{C.28}$$

One can treat the case $\alpha_1,\alpha_2<0$ in the same way and show that eq. (C.28) also holds in this case.

We would like to comment on the calculations in [17]. In [17], essentially a quantity such as

$$\int d'1d'2d'3\,\langle V_3(1,2,3)|\bar{\pi}_0^{(r)}|B_0\rangle_1^\epsilon|B_0\rangle_2^\epsilon|B_0\rangle_3^\epsilon\,, \tag{C.29}$$

is calculated to express the BRST transformation of the solitonic operators in terms of the string fields expanded by the normalized boundary state. Here let us consider the situation $\alpha_1\alpha_2 > 0$. This quantity can be calculated using either $\langle V_2(1,2);\epsilon|$ or $\langle V_1(3);\epsilon|$ by taking overlaps with $|B_0\rangle^\epsilon$'s.[4] Here let us devote our attention to the effect of the insertion of $C(\rho_I)$ in $\langle V_3(1,2,3)|$. From eq. (B.25) we can see that with ϵ small,

$$C(\rho_I) \sim \left(\frac{i}{\alpha_1}\pi_0^{(1)} + \frac{i}{\alpha_2}\pi_0^{(2)}\right)\ . \tag{C.30}$$

On the other hand, from eq. (C.27) one can see that

$$C(\rho_I) \sim \frac{2i}{\alpha_3}\pi_0^{(3)}\ . \tag{C.31}$$

Therefore the effect of inserting $C(\rho_I)$ is a bit asymmetric among 1st, 2nd and 3rd strings depending on the sign of α_r. These effects were overlooked in [17], and we took $C(\rho_I) \sim \frac{i}{\alpha_3}\pi_0^{(3)}$ instead of eq. (C.31), to calculate eq. (C.29). With these effects taken into account, the results in [17] are consistent with the ones here.

References

[1] A. Sen, *Universality of the tachyon potential*, *JHEP* **12** (1999) 027 [hep-th/9911116].

[2] J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *JHEP* **04** (2000) 012 [hep-th/0002117].

[3] R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, *Lumps and p-branes in open string field theory*, *Phys. Lett.* **B 482** (2000) 249 [hep-th/0003031].

[4] N. Moeller, A. Sen and B. Zwiebach, *D-branes as tachyon lumps in string field theory*, *JHEP* **08** (2000) 039 [hep-th/0005036].

[5] W. Siegel, *Covariantly second quantized string*, *Phys. Lett.* **B 142** (1984) 276.

[6] A. Neveu and P.C. West, *String lengths in covariant string field theory and OSp(26, 2/2)*, *Nucl. Phys.* **B 293** (1987) 266.

[7] S. Uehara, *On the covariantized light cone string field theory*, *Phys. Lett.* **B 190** (1987) 76; *On the covariantized light cone string field theory.* 2, *Phys. Lett.* **B 196** (1987) 47.

[8] T. Kugo, *Covariantized light cone string field theory*, in *Quantum mechanics of fundamental systems* 2 ed. C. Teitelboim and J. Zanelli, Plenum Publishing Corporation, Chap. 11 (1989).

[9] T. Kawano, *Dilaton condensation in the covariantized light cone closed string field theory*, *Prog. Theor. Phys.* **88** (1992) 1181.

[4]Notice that one should not use eqs. (B.26)(C.28) to calculate eq. (C.29). Eqs. (B.26)(C.28) hold in the leading order in ϵ and we have $\mathcal{O}\left(\frac{-1}{\ln\epsilon}\right)$ corrections.

[10] M. Kaku and K. Kikkawa, *The field theory of relativistic strings, pt. 1: trees*, *Phys. Rev.* **D 10** (1974) 1110;*The field theory of relativistic strings, pt. 2: loops and pomerons*, *Phys. Rev.* **D 10** (1974) 1823.

[11] S. Mandelstam, *Interacting string picture of dual resonance models*, *Nucl. Phys.* **B 64** (1973) 205.

[12] E. Cremmer and L. Gervais, *Infinite component field theory of interacting relativistic strings and dual theory*, *Nucl. Phys.* **B 90** (1975) 410.

[13] G. Parisi and Y.s. Wu, *Perturbation theory without gauge fixing*, *Sci. Sin.* **24** (1981) 483.

[14] G. Parisi and N. Sourlas, *Random magnetic fields, supersymmetry and negative dimensions*, *Phys. Rev. Lett.* **43** (1979) 744; *Supersymmetric field theories and stochastic differential equations*, *Nucl. Phys.* **B 206** (1982) 321.

[15] Y. Baba, N. Ishibashi and K. Murakami, *Observables and correlation functions in osp invariant string field theory*, *JHEP* **05** (2007) 020 [hep-th/0703216].

[16] T. Okuda and T. Takayanagi, *Ghost D-branes*, *JHEP* **03** (2006) 062 [hep-th/0601024].

[17] Y. Baba, N. Ishibashi and K. Murakami, *D-branes and closed string field theory*, *JHEP* **05** (2006) 029 [hep-th/0603152].

[18] N. Ishibashi and H. Kawai, *String field theory of noncritical strings*, *Phys. Lett.* **B 314** (1993) 190 [hep-th/9307045];
A. Jevicki and J.P. Rodrigues, *Loop space hamiltonians and field theory of noncritical strings*, *Nucl. Phys.* **B 421** (1994) 278 [hep-th/9312118].

[19] M. Fukuma and S. Yahikozawa, *Nonperturbative effects in noncritical strings with soliton backgrounds*, *Phys. Lett.* **B 396** (1997) 97 [hep-th/9609210]; *Combinatorics of solitons in noncritical string theory*, *Phys. Lett.* **B 393** (1997) 316 [hep-th/9610199].

[20] M. Hanada et al., *Loops versus matrices: the nonperturbative aspects of noncritical string*, *Prog. Theor. Phys.* **112** (2004) 131 [hep-th/0405076].

[21] I. Kishimoto, Y. Matsuo and E. Watanabe, *Boundary states as exact solutions of (vacuum) closed string field theory*, *Phys. Rev.* **D 68** (2003) 126006 [hep-th/0306189]; *A universal nonlinear relation among boundary states in closed string field theory*, *Prog. Theor. Phys.* **111** (2004) 433 [hep-th/0312122].

[22] J. Dai, R.G. Leigh and J. Polchinski, *New connections between string theories*, *Mod. Phys. Lett.* **A 4** (1989) 2073;
R.G. Leigh, *Dirac-Born-Infeld action from Dirichlet sigma model*, *Mod. Phys. Lett.* **A 4** (1989) 2767.

[23] J. Polchinski, S. Chaudhuri and C.V. Johnson, *Notes on D-branes*, hep-th/9602052;
J. Polchinski, *String theory. vol. 1: an introduction to the bosonic string*, Cambrige University Press, Cambridge Monographs on Mathematical Physics, Cambridge U.K. (1998).

[24] W. Siegel and B. Zwiebach, *Gauge string fields from the light cone*, *Nucl. Phys.* **B 282** (1987) 125; *Closed string fields via* $IOSp(1,1/2)$, *Phys. Lett.* **B 184** (1987) 325; $OSp(1,1/2)$ *string field theory with the world sheet metric*, *Nucl. Phys.* **B 288** (1987) 332;
L. Baulieu, W. Siegel and B. Zwiebach, *New coordinates for string fields*, *Nucl. Phys.* **B 287** (1987) 93.

[25] A.K.H. Bengtsson and N. Linden, *Interacting covariant open bosonic strings from the light cone* J^{I-}, *Phys. Lett.* **B 187** (1987) 289.

[26] A. LeClair, M.E. Peskin and C.R. Preitschopf, *String field theory on the conformal plane.* 1. *Kinematical principles*, *Nucl. Phys.* **B 317** (1989) 411.

[27] A. LeClair, M.E. Peskin and C.R. Preitschopf, *String field theory on the conformal plane.* 2. *Generalized gluing*, *Nucl. Phys.* **B 317** (1989) 464;
T. Asakawa, T. Kugo and T. Takahashi, *On the generalized gluing and resmoothing theorem*, *Prog. Theor. Phys.* **100** (1998) 437 [hep-th/9805119].

[28] S. Mandelstam, *The interacting string picture and functional integration*, in *workshop on unified string theories,* eds. M.B. Green and D.J. Gross, World Scientific, Singapore (1986).

[29] I. Kishimoto and Y. Matsuo, *Cardy states, factorization and idempotency in closed string field theory*, *Nucl. Phys.* **B 707** (2005) 3 [hep-th/0409069].

[30] T. Asakawa, T. Kugo and T. Takahashi, *One-loop tachyon amplitude in unoriented open-closed string field theory*, *Prog. Theor. Phys.* **102** (1999) 427 [hep-th/9905043].

[31] I. Kishimoto, S. Moriyama and S. Teraguchi, *Twist field as three string interaction vertex in light cone string field theory*, *Nucl. Phys.* **B 744** (2006) 221 [hep-th/0603068].

[32] S.B. Giddings and S.A. Wolpert, *A triangulation of moduli space from light cone string theory*, *Commun. Math. Phys.* **109** (1987) 177;
K. Ito and T. Onogi, *One loop integration region in closed light cone string field theory*, *Prog. Theor. Phys.* **78** (1987) 135.